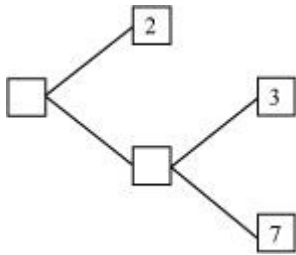


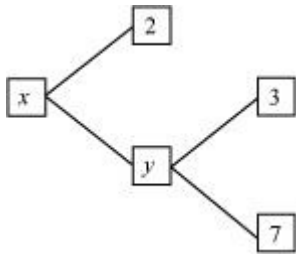
Question 1 ( 1.0 marks)

Complete the missing entries in the following factor tree:



Solution:

Let the missing numbers be  $x$  and  $y$ .

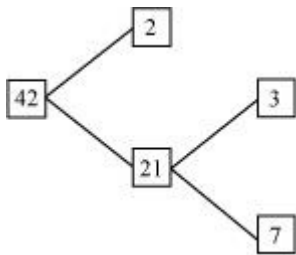


From the above factor tree, it is clear that:

$$y = 3 \times 7 = 21$$

$$\text{And, } x = 2 \times y = 2 \times 21 = 42$$

□



Question 2 ( 1.0 marks)

If  $(x + a)$  is a factor of  $2x^2 + 2ax + 5x + 10$ , find  $a$ .

Solution:

$$\begin{array}{r}
 \phantom{x+a} \overline{2x+5} \\
 x+a \overline{) 2x^2 + 2ax + 5x + 10} \\
 \underline{2x^2 + 2ax} \phantom{+ 5x + 10} \\
 \phantom{2x^2 + 2ax} 5x + 10 \\
 \phantom{2x^2 + 2ax} \underline{5x + 5a} \\
 \phantom{2x^2 + 2ax} \phantom{5x + 10} \phantom{5x + 5a} \underline{\phantom{0} - \phantom{0}} \\
 \phantom{2x^2 + 2ax} \phantom{5x + 10} \phantom{5x + 5a} 10 - 5a
 \end{array}$$

If  $x + a$  is a factor of  $2x^2 + 2ax + 5x + 10$ , then the remainder i.e.,  $10 - 5a = 0 \Rightarrow a = 2$ .

#### Question 3 ( 1.0 marks)

Show that  $x = -3$  is a solution of  $x^2 + 6x + 9 = 0$ .

Solution:

$$\text{L.H.S} = x^2 + 6x + 9 = (-3)^2 + 6(-3) + 9 \quad (\because x = -3)$$

$$= 9 - 18 + 9$$

$$= 0 = \text{R.H.S}$$

This proves that  $x = -3$  is a solution of  $x^2 + 6x + 9 = 0$ .

#### Question 4 ( 1.0 marks)

The first term of an A.P. is  $p$  and its common difference is  $q$ . Find its 10<sup>th</sup> term.

Solution:

We know that the  $n$ th term of an A.P with the first term  $a$  and common difference  $d$  is given by:

$$a_n = a + (n-1)d$$

$$\begin{aligned}
 \therefore a_{10} &= p + (10-1)q && (\because a = p, d = q, n = 10) \\
 &= p + 9q
 \end{aligned}$$

#### Question 5 ( 1.0 marks)

If  $\tan A = \frac{5}{12}$ , find the value of  $(\sin A + \cos A) \sec A$ .

Solution:

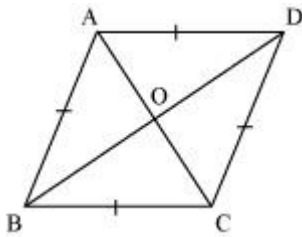
$$\begin{aligned}(\sin A + \cos A) \sec A &= (\sin A + \cos A) \cdot \frac{1}{\cos A} \\ &= \frac{\sin A}{\cos A} + 1 \\ &= \tan A + 1 \\ &= \frac{5}{12} + 1 = \frac{17}{12}\end{aligned}$$

Question 6 ( 1.0 marks)

The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.

Solution:

Consider a rhombus ABCD.



We know that the diagonals of a rhombus bisect each other at right angles.

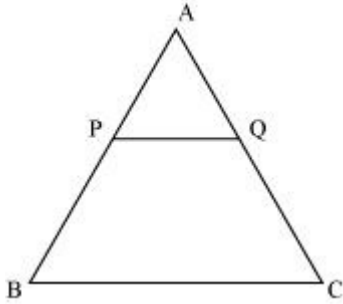
Now, using the Pythagoras theorem for  $\triangle BOC$ , we get

$$\begin{aligned}BC^2 &= OB^2 + OC^2 = \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 = [(20)^2 + (15)^2] \text{ cm}^2 \\ &= 625 \text{ cm}^2\end{aligned}$$

$\therefore$  Side of rhombus =  $BC = 25 \text{ cm}$

Question 7 ( 1.0 marks)

In the figure given below,  $PQ \parallel BC$  and  $AP:PB = 1:2$ . Find  $\frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)}$ .



Solution:

Since  $AP:PB = 1:2$ ,

$$\frac{AP}{PB} = \frac{1}{2} \Rightarrow PB = 2AP$$

In  $\triangle APQ$  and  $\triangle ABC$ ,

$$\square APQ = \square ABC \text{ (PQ} \parallel \text{BC)}$$

$$\square PAQ = \square BAC \text{ (Common angle)}$$

$$\square \triangle APQ \sim \triangle ABC \text{ (AA similarity)}$$

$$\Rightarrow \frac{\text{area}(\triangle APQ)}{\text{area}(\triangle ABC)} = \frac{AP^2}{AB^2} = \frac{AP^2}{(AP+PB)^2} = \frac{AP^2}{(3AP)^2} = \frac{1}{9}$$

Question 8 ( 1.0 marks)

The surface area of a sphere is  $616 \text{ cm}^2$ . Find its radius.

Solution:

$$\text{Surface area of sphere} = 4\pi r^2 = 616 \text{ cm}^2$$

$$\Rightarrow r^2 = \left( \frac{616 \times 7}{4 \times 22} \right) \text{ cm}^2 = 49 \text{ cm}^2$$

$$\square \text{ Radius} = r = 7 \text{ cm}$$

Question 9 ( 1.0 marks)

A dice is thrown once. Find the probability of getting a number less than 3.

Solution:

When a dice is thrown once, then the sample space = {1, 2, 3, 4, 5, 6}

This means that there are 6 observations in the sample space.

∴ Observations less than 3 = {1, 2}

Thus, there are 2 observations which can give us a number less than 3.

$$\square P(\text{number less than 3}) = \frac{2}{6} = \frac{1}{3}$$

Question 10 ( 1.0 marks)

Find the class marks of classes 10 – 25 and 35 – 55.

Solution:

$$\text{Class mark} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

$$\square \text{ Class marks of } 10 - 25 = \frac{10 + 25}{2} = 17.5$$

$$\text{And, class marks of } 35 - 55 = \frac{35 + 55}{2} = 45$$

## Section B

Question number 11 to 15 carry 2 marks each.

Question 11 ( 2.0 marks)

Find all the zeroes of the polynomial  $x^4 + x^3 - 34x^2 - 4x + 120$ , if two of its zeros are 2 and -2.

Solution:

Since 2 and -2 are the two zeroes of  $x^4 + x^3 - 34x^2 - 4x + 120$ , then  $(x - 2)(x + 2) = x^2 - 4$  is a factor of  $x^4 + x^3 - 34x^2 - 4x + 120$ .



In  $\Delta ABC$  right-angled at C, if  $\tan A = \frac{1}{\sqrt{3}}$ , then find the value of  $\sin A \cos B + \cos A \sin B$ .

Solution:

$$\begin{aligned}\sec 4A &= \operatorname{cosec}(A - 20^\circ) \\ &= \sec[90^\circ - (A - 20^\circ)] \\ \therefore 4A &= 90^\circ - (A - 20^\circ) = 110^\circ - A \\ \Rightarrow A &= 22^\circ\end{aligned}$$

OR,

$$\begin{aligned}\tan A &= \frac{1}{\sqrt{3}} = \tan 30^\circ \\ \Rightarrow \angle A &= 30^\circ\end{aligned}$$

Since  $\angle C = 90^\circ$ ,

$$\angle A + \angle B = 90^\circ$$

$$\angle B = 60^\circ$$

Now,

$$\begin{aligned}\sin A \cos B + \cos A \sin B &= \sin 30^\circ \cdot \cos 60^\circ + \cos 30^\circ \cdot \sin 60^\circ \\ &= \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{1}{4} + \frac{3}{4} \\ &= 1\end{aligned}$$

#### Question 14 ( 2.0 marks)

Find the value of  $k$  if the points  $(k, 3)$ ,  $(6, -2)$ , and  $(-3, 4)$  are collinear.

Solution:

The points  $(k, 3)$ ,  $(6, -2)$ , and  $(-3, 4)$  are collinear, if the area of the triangle formed through these points is zero.

i.e.,

$$\frac{1}{2}|k(-2-4)+6(4-3)-3(3+2)|=0$$

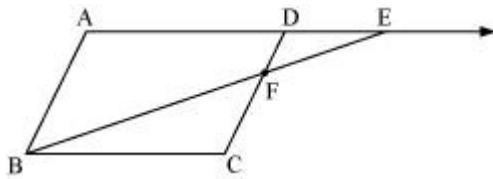
$$\Rightarrow -6k-9=0$$

$$\Rightarrow k = \frac{-3}{2}$$

Question 15 ( 2.0 marks)

E is a point on AD produced of a  $\parallel^{\text{gm}}$  ABCD and BE intersects CD at F. Show that  $\Delta ABE \sim \Delta CFB$ .

Solution:



In  $\Delta ABE$  and  $\Delta CFB$

$\angle BAE = \angle FCB$  (Opposite angles of  $\parallel^{\text{gm}}$  ABCD)

$\angle AEB = \angle CBF$  (Alternate interior angles)

$\therefore \Delta ABE \sim \Delta CFB$  (AA similarity)

## Section C

Question number 16 to 25 carry 3 marks each.

Question 16 ( 3.0 marks)

Use Euclid's Division Lemma to show that the square of any positive integer is either of the form  $3m$  or  $(3m + 1)$  for some integer  $m$ .

Solution:

Let  $a$  be any positive integer and  $b = 3$ .

Then,



$$a = 3q + r \text{ (For some integers } q > 0)$$

And,

$$r = 0, 1, 2 \text{ (Since } 0 \leq r < 3)$$

$$\therefore a = 3q \text{ or } 3q + 1 \text{ or } 3q + 2$$

Now,

$$a^2 = (3q)^2 \text{ or } (3q + 1)^2 \text{ or } (3q + 2)^2$$

$$\square a^2 = 9q^2 \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4$$

$$\square a^2 = 3(3q^2) \text{ or } 3(3q^2 + 2q) + 1 \text{ or } 3(3q^2 + 4q + 1) + 1$$

$$\square a^2 = 3k_1 \text{ or } 3k_2 + 1 \text{ or } 3k_3 + 1, \text{ where } k_1, k_2, \text{ and } k_3 \text{ are some positive integers.}$$

Hence, it has been shown that the square of any positive integer is either of the form  $3m$  or  $3m + 1$ .

#### Question 17 ( 3.0 marks)

Represent the following pair of equations graphically and write the co-ordinates of points where the lines intersect the y-axis:

$$x + 3y = 6$$

$$2x - 3y = 12$$

Solution:

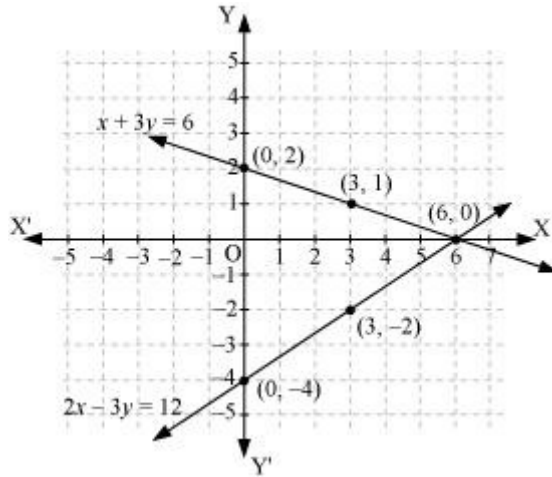
For the linear equation  $x + 3y = 6$  i.e.,  $x = 6 - 3y$ , the solution table is as follows:

$x$	6	3	0
$y$	0	1	2

For the linear equation  $2x - 3y = 12$  i.e.,  $x = \frac{12 + 3y}{2}$ , the solution table is as follows:

$X$	6	3	0
$y$	0	-2	-4

The graphical representation of the given linear equations is as follows:



Hence, the co-ordinates of points where the line,  $x + 3y = 6$  and the line,  $2x - 3y = 12$  intersect the y-axis are  $(0, 2)$  and  $(0, -4)$  respectively.

Question 18 ( 3.0 marks)

For what value of  $n$  are the  $n^{\text{th}}$  terms of two A.P.'s  $63, 65, 67, \dots$  and  $3, 10, 17, \dots$  equal?

**OR**

If  $m$  times the  $m^{\text{th}}$  term of an A.P. is equal to  $n$  times its  $n^{\text{th}}$  term, then find the  $(m + n)^{\text{th}}$  term of the A.P.

Solution:

For A.P,  $63, 65, 67, \dots$

$$a = 63, d = 65 - 63 = 2$$

$$\square a_n = a + (n - 1) d = 63 + (n - 1) 2 = 2n + 61$$

For AP,  $3, 10, 17, \dots$

$$a = 3, d = 10 - 3 = 7$$

$$\square a_n = a + (n - 1) d = 3 + (n - 1)7 = 7n - 4$$

Since the  $n^{\text{th}}$  terms of the two A.P.'s are equal,

$$2n + 61 = 7n - 4$$

$$\square 5n = 65$$

$$\square n = 13$$

OR,

Let the first term and common difference of A.P.'s be  $a$  and  $d$  respectively.

Now,

$$m \cdot a_m = n \cdot a_n$$

$$\Rightarrow m \cdot [a + (m-1)d] = n \cdot [a + (n-1)d]$$

$$\Rightarrow (m-n)a + (m^2 - m - n^2 + n)d = 0$$

$$\Rightarrow (m-n)a + [(m-n)(m+n) - (m-n)]d = 0$$

$$\Rightarrow (m-n)[a + (m+n-1)d] = 0$$

$$\Rightarrow a + (m+n-1)d = 0$$

$$\Rightarrow a_{m+n} = 0$$

#### Question 19 ( 3.0 marks)

In an A.P. the first term is 8, the  $n^{\text{th}}$  term is 33, and sum of the first  $n$  terms is 123. Find  $n$  and  $d$ , the common difference.

Solution:

In the given A.P,

$$a = 8$$

And,

$$a_n = a + (n-1)d = 33$$

$$\Rightarrow 8 + (n-1)d = 33$$

$$\therefore (n-1)d = 25 \quad \dots(i)$$

It is also given that,

$$S_n = \frac{n}{2}[2a + (n-1)d] = 123$$

$$\Rightarrow \frac{n}{2}[2 \times 8 + 25] = 123$$

$$\Rightarrow n = 6$$

Substituting  $n = 6$  in equation (i), we obtain

$$5d = 25 \quad \square \quad d = 5$$

$$\square \quad d = 5 \text{ and } n = 6$$

Question 20 ( 3.0 marks)

Prove that:

$$(1 + \cot A + \tan A) (\sin A - \cos A) = \sin A \tan A - \cot A \cos A$$

**OR**

Without using trigonometric tables, evaluate the following:

$$2 \left( \frac{\cos 58^\circ}{\sin 32^\circ} \right) - \sqrt{3} \left( \frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ} \right)$$

Solution:

$$\text{L.H.S} = (1 + \cot A + \tan A) (\sin A - \cos A)$$

$$= \left( 1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A} \right) (\sin A - \cos A)$$

$$= \left( \frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cos A} \right) (\sin A - \cos A)$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A} \quad \left\{ a^3 - b^3 = (a - b)(a^2 + b^2 + ab) \right\}$$

$$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

$$= \sin A \tan A - \cos A \cot A$$

$$= \text{R.H.S}$$

OR,

$$\begin{aligned}
& 2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left(\frac{\cos 38^\circ \cdot \operatorname{cosec} 52^\circ}{\tan 15^\circ \cdot \tan 60^\circ \cdot \tan 75^\circ}\right) \\
&= 2\left[\frac{\cos(90^\circ - 32^\circ)}{\sin 32^\circ}\right] - \sqrt{3}\left[\frac{\cos 38^\circ \cdot \operatorname{cosec}(90^\circ - 38^\circ)}{\tan(90^\circ - 75^\circ) \cdot \sqrt{3} \cdot \tan 75^\circ}\right] \\
&= 2\left(\frac{\sin 32^\circ}{\sin 32^\circ}\right) - \frac{\cos 38^\circ \cdot \sec 38^\circ}{\cot 75^\circ \cdot \tan 75^\circ} \\
&= 2 \times 1 - \frac{1}{1} \\
&= 1
\end{aligned}$$

#### Question 21 ( 3.0 marks)

If P divides the join of A(-2, -2) and B(2, -4) such that  $\frac{AP}{AB} = \frac{3}{7}$ , find the co-ordinates of P.

Solution:

$$\begin{aligned}
\therefore \frac{AP}{AB} &= \frac{3}{7} \\
\Rightarrow \frac{AP}{PB} &= \frac{3}{4}
\end{aligned}$$



Now, the co-ordinates of P can be given as:

$$\begin{aligned}
& \left( \frac{3 \times 2 + 4 \times (-2)}{3 + 4}, \frac{3 \times (-4) + 4 \times (-2)}{3 + 4} \right) \\
&= \left( \frac{-2}{7}, \frac{-20}{7} \right)
\end{aligned}$$

#### Question 22 ( 3.0 marks)

The mid-points of the sides of a triangle are (3, 4), (4, 6), and (5, 7). Find the co-ordinates of the vertices of the triangle.

Solution:

Let D(3, 4), E(4, 6), and F(5, 7) be the mid-points of sides AB, BC, and AC of  $\Delta ABC$  (respectively) formed by vertices  $A(x_1, y_1)$ ,  $B(x_2, y_2)$ , and  $C(x_3, y_3)$ .

It is given that D is the mid-point of AB.

$$\therefore \frac{x_1 + x_2}{2} = 3 \text{ and } \frac{y_1 + y_2}{2} = 4$$

$$\Rightarrow x_1 + x_2 = 6 \quad \dots(\text{i})$$

And,

$$y_1 + y_2 = 8 \quad \dots(\text{ii})$$

Similarly,

$$\frac{x_2 + x_3}{2} = 4, \frac{y_2 + y_3}{2} = 6, \frac{x_1 + x_3}{2} = 5, \frac{y_1 + y_3}{2} = 7$$

$$\Rightarrow x_2 + x_3 = 8 \quad \dots(\text{iii})$$

$$\Rightarrow y_2 + y_3 = 12 \quad \dots(\text{iv})$$

$$\Rightarrow x_1 + x_3 = 10 \quad \dots(\text{v})$$

$$\Rightarrow y_1 + y_3 = 14 \quad \dots(\text{vi})$$

Adding equations (i), (iii), and (v), we get:

$$2(x_1 + x_2 + x_3) = 24$$

$$\Rightarrow x_1 + x_2 + x_3 = 12 \quad \dots(\text{vii})$$

Similarly, adding equations (ii), (iv), and (vi), we obtain:

$$2(y_1 + y_2 + y_3) = 34$$

$$\Rightarrow y_1 + y_2 + y_3 = 17 \quad \dots(\text{viii})$$

Solving equations (i) and (vii), we get:

$$x_3 = 6$$

Solving equations (iii) and (vii), we get:

$$x_1 = 4$$

Solving equations (v) and (viii), we get:

$$x_2 = 2$$

Similarly, solving equations (ii), (iv), and (vi) with (viii), we get:

$$y_3 = 9, y_1 = 5, y_2 = 3$$

Hence, the co-ordinates of the vertices of the triangle are (4, 5), (2, 3), and (6, 9).

#### Question 23 ( 3.0 marks)

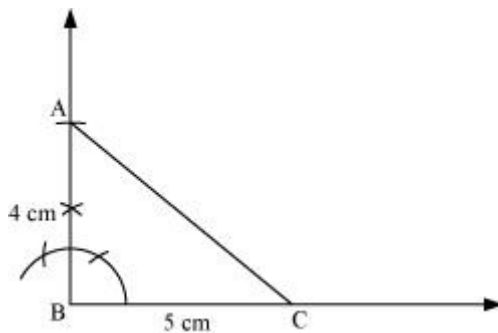
Draw a right triangle where the sides containing the right angle are 5 cm and 4 cm.

Construct a similar triangle whose sides are  $\frac{5}{3}$  times the sides of the above triangle.

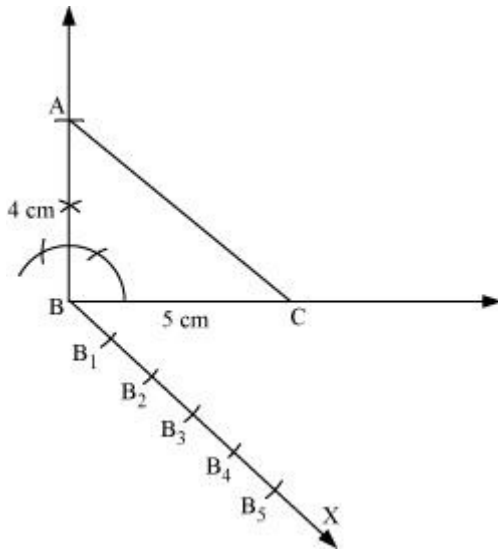
Solution:

#### Steps of construction:

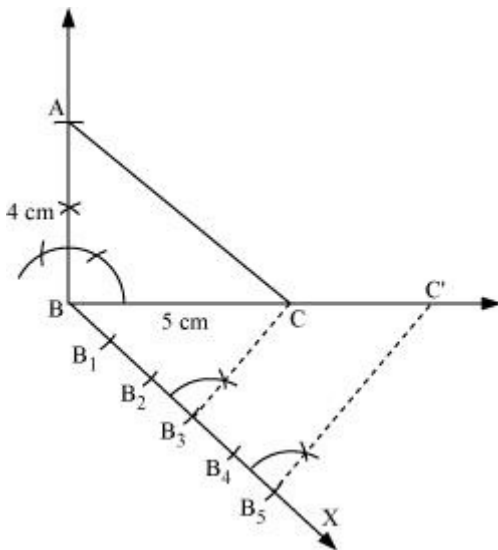
1. Draw a right triangle ABC right-angled at B such that AB = 4 cm and BC = 5 cm.



2. Draw a ray BX making an acute angle with BC on the side opposite to vertex A.  
On BX, locate 5 points B<sub>1</sub>, B<sub>2</sub>, B<sub>3</sub>, B<sub>4</sub>, and B<sub>5</sub> such that BB<sub>1</sub> = B<sub>1</sub>B<sub>2</sub> = B<sub>2</sub>B<sub>3</sub> = B<sub>3</sub>B<sub>4</sub> = B<sub>4</sub>B<sub>5</sub>.

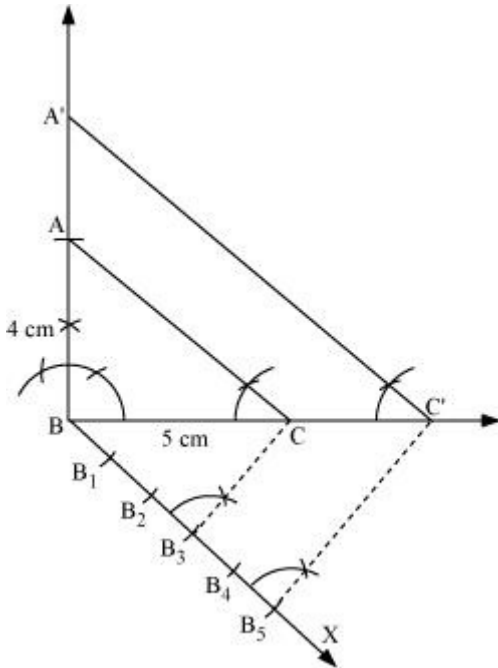


3. Join  $B_3C$ . Draw a line through  $B_5$  parallel to  $B_3C$ , intersecting ray  $BC$  at  $C'$ .



4. Draw a line through  $C'$  parallel to  $CA$ , intersecting ray  $BA$  at  $A'$ .





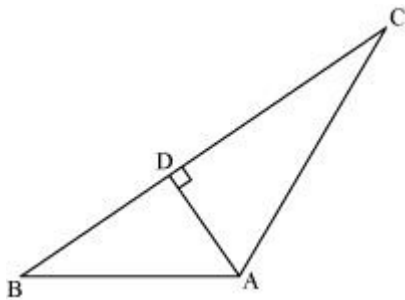
Hence,  $\Delta A'BC' \sim \Delta ABC$  where the sides of  $\Delta A'BC'$  are  $\frac{5}{3}$  times the sides of the right-angled triangle ABC.

Question 24 ( 3.0 marks)

Prove that a parallelogram circumscribing a circle is a rhombus.

**OR**

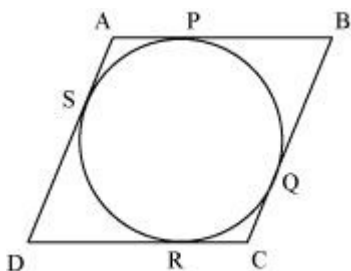
In Figure 2,  $AD \perp BC$ . Prove that  $AB^2 + CD^2 = BD^2 + AC^2$ .



Solution:

Consider that  $\square ABCD$  is circumscribed in a circle.

Sides AB, BC, CD, and AD touch the circle at P, Q, R, and S respectively.



It is known that the lengths of the tangents drawn from an external point to a circle are equal.

$$\square AP = AS \dots (i)$$

$$PB = BQ \dots (ii)$$

$$DR = DS \dots (iii)$$

$$CR = CQ \dots (iv)$$

Adding equations (i) to (iv), we obtain

$$(AP + PB) + (DR + RC) = (AS + DS) + (BQ + CQ)$$

$$\square AB + CD = AD + BC$$

$$\square 2AB = 2BC \quad (\because ABCD \text{ is a } \parallel^{\text{gm}}, AB = CD \text{ and } AD = BC)$$

$$\square AB = BC$$

Hence, ABCD is a rhombus since ABCD is a  $\parallel^{\text{gm}}$  and the adjacent sides are equal ( $AB = BC$ ).

OR,

Using the Pythagoras theorem for  $\triangle ABD$ , we obtain

$$AD^2 + BD^2 = AB^2$$

$$\square AD^2 = AB^2 - BD^2 \dots (i)$$

Using the Pythagoras theorem for  $\triangle ACD$ , we obtain

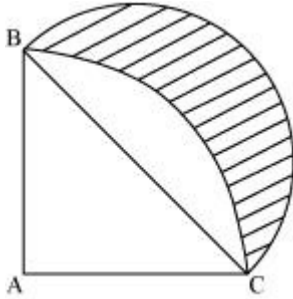
$$AD^2 + CD^2 = AC^2$$

$$\square AB^2 - BD^2 + CD^2 = AC^2 \text{ [Using equation (i)]}$$

$$\square AB^2 + CD^2 = AC^2 + BD^2$$

Question 25 ( 3.0 marks)

In Figure 3, ABC is the quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as the diameter. Find the area of the shaded region.



Solution:

Since ABC is a quadrant of the circle of radius 14 cm,

$$AB = AC = 14 \text{ cm}$$

And,

$$\square \angle BAC = 90^\circ$$

$\square$  Using the Pythagoras theorem for  $\triangle BAC$ , we obtain

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{(14\text{cm})^2 + (14\text{cm})^2} = 14\sqrt{2} \text{ cm}$$

Now,

Area of shaded region = Area of semi-circle taking BC as diameter

– (Area of quadrant ABC – Area of  $\triangle ABC$ )

$$\begin{aligned} &= \left[ \frac{1}{2} \pi \times (7\sqrt{2})^2 - \left\{ \frac{1}{4} \times \pi \times (14)^2 - \frac{1}{2} \times 14 \times 14 \right\} \right] \\ &= [154 - (154 - 98)] \text{ cm}^2 \\ &= 98 \text{ cm}^2 \end{aligned}$$

## Section D

Question number 26 to 30 carry 6 marks each.

Question 26 ( 6.0 marks)

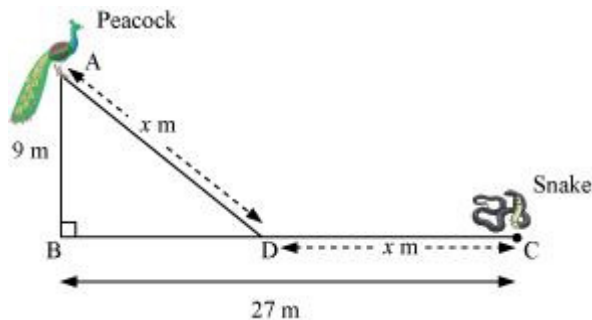
A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?

OR

The difference of two numbers is 4. If the difference of their reciprocals is  $\frac{4}{21}$ , find the two numbers.

Solution:

Since the speed of the peacock and snake is equal, they will cover an equal distance in the same time limit. Then, let each animal cover  $x$  m. The situation can be represented with the help of a figure as below:



Here, AD and DC represent the distance covered by the peacock and the snake respectively.

Using the Pythagoras theorem for right triangle ABD, we obtain:

$$AB^2 + BD^2 = AD^2$$

$$\Rightarrow (9)^2 + (27 - x)^2 = x^2$$

$$\Rightarrow 81 + 729 + x^2 - 54x = x^2$$

$$\Rightarrow 54x = 810$$

$$\Rightarrow x = 15$$

Now,  $CD = x \text{ m} = 15 \text{ m}$

And,

$$BD = (27 - x) \text{ m} = (27 - 15) \text{ m} = 12 \text{ m}$$

Hence, the snake is caught at a distance of 12 m from the hole.

OR,

Let the smaller number be  $x$  and the larger number be  $x + 4$ .

It is given that,

$$\frac{1}{x} - \frac{1}{x+4} = \frac{4}{21}$$

$$\Rightarrow \frac{4}{x(x+4)} = \frac{4}{21}$$

$$\Rightarrow x^2 + 4x - 21 = 0$$

$$\Rightarrow x^2 + 7x - 3x - 21 = 0$$

$$\Rightarrow (x+7)(x-3) = 0$$

$$\Rightarrow x = 3 \text{ or } -7$$

If  $x = 3$ , then  $x + 4 = 7$ .

If  $x = -7$ , then  $x + 4 = -3$ .

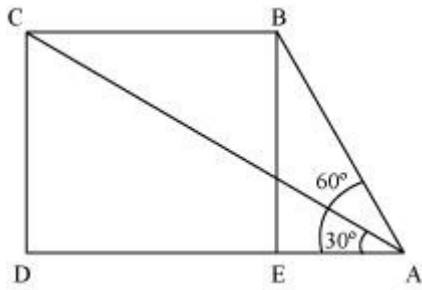
Hence, the numbers are 3 and 7 or  $-7$  and  $-3$ .

#### Question 27 ( 6.0 marks)

The angle of elevation of an aeroplane from a point A on the ground is  $60^\circ$ . After a flight of 30 seconds, the angle of elevation changes to  $30^\circ$ . If the plane is flying at a constant height of  $3600\sqrt{3}$  m, find the speed in km/hour of the plane.

Solution:

Let B and C be the initial and final positions of the aeroplane respectively.



Height of the aeroplane from the ground,  $BE = CD = 3600\sqrt{3}$  m

In  $\triangle AEB$ ,

$$\frac{BE}{AE} = \tan 60^\circ$$

$$\Rightarrow \frac{3600\sqrt{3} \text{ m}}{AE} = \sqrt{3}$$

$$\Rightarrow AE = 3600 \text{ m}$$

In  $\triangle ACD$ ,

$$\frac{CD}{AD} = \tan 30^\circ$$

$$\Rightarrow \frac{3600\sqrt{3} \text{ m}}{AD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AD = 10800 \text{ m}$$

Distance travelled by the aeroplane in 30 s = BC

$$= AD - AE$$

$$= (10800 - 3600) \text{ m}$$

$$= 7200 \text{ m}$$

$$\square \text{ Speed of the aeroplane} = \frac{\text{Distance}}{\text{Time}}$$

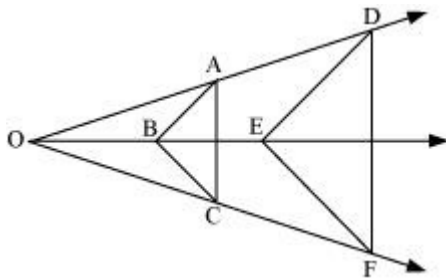
$$\begin{aligned}
&= \frac{7200 \text{ m}}{30 \text{ s}} \\
&= \frac{\left(\frac{7200}{1000}\right) \text{ km}}{\left(\frac{30}{60 \times 60}\right) \text{ hr}} \\
&= \left(\frac{72 \times 120}{10}\right) \text{ km/hr} \\
&= 864 \text{ km/hr}
\end{aligned}$$

Question 28 ( 6.0 marks)

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Using the above, prove the following:

In Figure 4,  $AB \parallel DE$  and  $BC \parallel EF$ . Prove that  $AC \parallel DF$ .

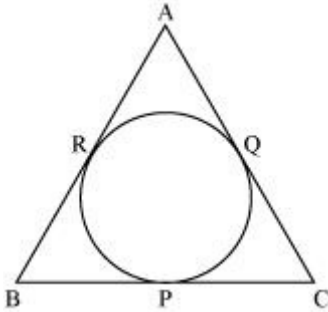


**OR**

Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Using the above, prove the following:

$ABC$  is an isosceles triangle in which  $AB = AC$  circumscribed about a circle, as shown in Figure 5. Prove that the base is bisected by the point of contact.



Solution:

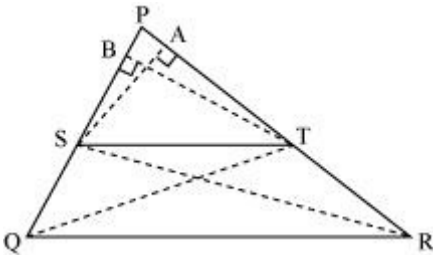
**Part I:**

Let us consider a  $\Delta PQR$  in which  $ST \parallel QR$ , where  $ST$  intersects  $PQ$  and  $PR$  at  $S$  and  $T$  respectively.

$$\frac{PS}{QS} = \frac{PT}{RT}$$

We can prove the given statement, if we can prove

For this, let us join  $RS$  and  $QT$  and draw  $SA \perp PR$  and  $TB \perp PQ$ .



$$\text{Now, } \frac{\text{area}(\Delta PST)}{\text{area}(\Delta SQT)} = \frac{\frac{1}{2} \times PS \times BT}{\frac{1}{2} \times QS \times BT} = \frac{PS}{QS} \quad \dots(i)$$

$\left( \because \text{Area of a triangle} = \frac{1}{2} \text{base} \times \text{altitude} \right)$

$$\text{Similarly, } \frac{\text{area}(\Delta PST)}{\text{area}(\Delta SRT)} = \frac{\frac{1}{2} \cdot PT \cdot AS}{\frac{1}{2} RT \cdot AS} = \frac{PT}{RT} \quad \dots(ii)$$

$\Delta SQT$  and  $\Delta SRT$  are on the same base ( $ST$ ) and between the same parallels ( $ST$  and  $QR$ ).

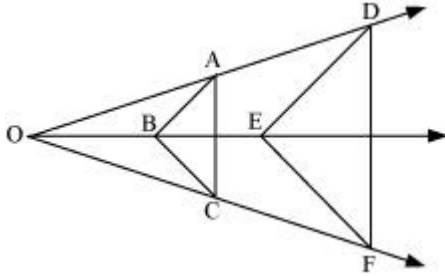
$\square$   $\text{Area}(\Delta SQT) = \text{Area}(\Delta SRT) \dots(iii)$



On comparing equations (i), (ii), and (iii), we obtain:

$$\frac{PS}{QS} = \frac{PT}{RT}$$

**Part II:**



In  $\triangle OED$ ,  $AB \parallel DE$

$$\Rightarrow \frac{OA}{AD} = \frac{OB}{BE} \quad \dots(\text{iv})$$

In  $\triangle OEF$ ,  $BC \parallel EF$

$$\Rightarrow \frac{OB}{BE} = \frac{OC}{CF} \quad \dots(\text{v})$$

Comparing equations (iv) and (v), we get

$$\frac{OA}{AD} = \frac{OC}{CF}$$

□  $AC \parallel DF$  (converse of B.P.T)

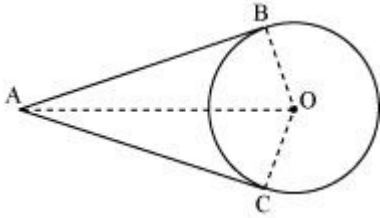
OR,

**Part I:**

Let us consider a circle with centre O. Let point A be lying outside the circle. Let AB and AC be the tangents drawn to the circle from point A.

We can prove the given statement, if we can prove  $AB = AC$ .

For this, let us join OA, OB, and OC.



In  $\triangle OAB$  and  $\triangle OAC$ ,

$OA = OA$  (Common)

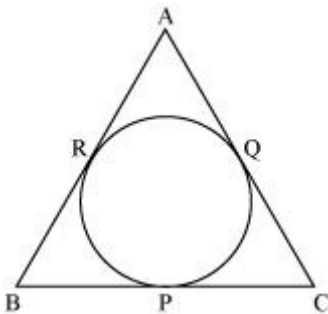
$OB = OC$  (Radii of the same circle)

$\angle OBA = \angle OCA = 90^\circ$  (Radius  $\perp$  tangent)

$\triangle OAB \cong \triangle OAC$  (R.H.S congruence)

$AB = AC$  (CPCT)

**Part II:**



In the given figure, A is a point lying outside the circle. Let AR and AQ be the tangents drawn to the circle.

$AR = AQ \dots(i)$

Similarly,  $BP = BR \dots(ii)$

And,  $CP = CQ \dots(iii)$

$AB = AC \dots(iv)$  (Given)

Subtracting equation (i) from equation (iv), we obtain:

$$AB - AR = AC - AQ$$

$$\square BR = CQ$$

$$\square BP = CP \text{ [Using equations (ii) and (iii)]}$$

Question 29 ( 6.0 marks)

If the radii of the circular ends of a conical bucket, which is 16 cm high, are 20 cm and 8 cm, find the capacity and the total surface area of the bucket. [Use  $\pi = \frac{22}{7}$  ]

Solution:

The radii of the circular ends of the conical bucket (having the shape of a frustum of a cone) are:

$$r_1 = 20 \text{ cm}, r_2 = 8 \text{ cm}$$

$$\text{Height, } h = 16 \text{ cm}$$

Now,

$$\begin{aligned} l &= \sqrt{h^2 + (r_1 - r_2)^2} \\ &= \left[ \sqrt{(16)^2 + (20 - 8)^2} \right] \text{ cm} \\ &= 20 \text{ cm} \end{aligned}$$

$$\text{Capacity of the bucket} = \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$$

$$\begin{aligned} &= \left[ \frac{1}{3} \times \frac{22}{7} \times 16 (20^2 + 8^2 + 20 \times 8) \right] \text{ cm}^3 \\ &= \frac{73216}{7} \text{ cm}^3 \\ &= 10459.43 \text{ cm}^3 \end{aligned}$$

Total surface area of the bucket (circular base of radius 20 cm is open):

$$\begin{aligned}
&= \pi [(r_1 + r_2)l + r_2^2] \\
&= \frac{22}{7} [(20 + 8) \times 20 + (8)^2] \text{ cm}^2 \\
&= \left( \frac{13728}{7} \right) \text{ cm}^2 \\
&= 1961.14 \text{ cm}^2
\end{aligned}$$

Question 30 ( 6.0 marks)

Find mean, median, and mode of the following data:

Classes	Frequency
0 – 20	6
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	6
100 – 120	5
120 – 140	3

Solution:

Class interval	Frequency ( $f_i$ )	Class marks $x_i$	$f_i x_i$
0 – 20	6	10	60
20 – 40	8	30	240
40 – 60	10	50	500
60 – 80	12	70	840
80 – 100	6	90	540
100 – 120	5	110	550
120 – 140	3	130	390
	$\sum f_i = 50$		$\sum f_i x_i = 3120$

$$\text{Mean, } \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3120}{50} = 62.4$$

Class interval 60 – 80 has maximum frequency. Now, the modal class is 60 – 80.

□  $l$  = lower limit of the modal class = 60

$h$  = size of the modal class = 80 – 60 = 20

$f_1$  = frequency of the modal class = 12

$f_0$  = frequency of the class preceding the modal class = 10

$f_2$  = frequency of the class succeeding the modal class = 6

$$\begin{aligned} \therefore \text{Mode} &= l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 60 + \left( \frac{12 - 10}{24 - 10 - 6} \right) \times 20 \\ &= 60 + 5 \\ &= 65 \end{aligned}$$

To find the median, we have to calculate the cumulative frequency of each class interval as shown below:

Class interval	$f$	$cf$
0 – 20	6	6
20 – 40	8	14
40 – 60	10	24
60 – 80	12	36
80 – 100	6	42
100 – 120	5	47
120 – 140	3	50
	$n = 50$	

$$\text{Now, } \frac{n}{2} = \frac{50}{2} = 25$$

The 25<sup>th</sup> observation lies in class 60 – 80.

∴ Median class = 60 – 80

□  $l$  = lower limit of the median class = 60

$cf$  = cumulative frequency of the class preceding the median class = 24

$f$  = frequency of the median class = 12

$h$  = class size = 20

$$\begin{aligned}\therefore \text{Median} &= l + \left[ \frac{\frac{n}{2} - cf}{f} \right] \times h \\ &= 60 + \left( \frac{25 - 24}{12} \right) \times 20 \\ &= 60 + \frac{5}{3} \\ &= 61.67\end{aligned}$$