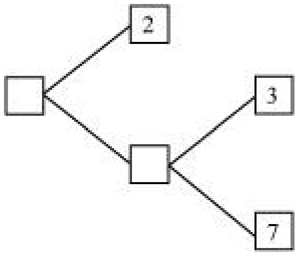
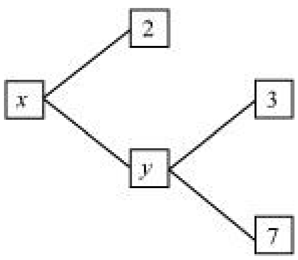
Question 1 (1.0 marks)

Complete the missing entries in the following factor tree:



Solution:

Let the missing numbers be x and y.

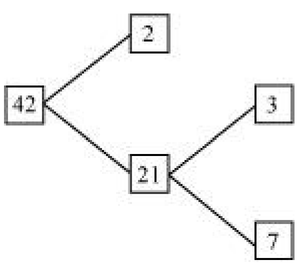


From the above factor tree, it is clear that:

$y = 3 \times 7 = 21$

And, $x = 2 \times y = 2 \times 21 = 42$

□



Question 2 (1.0 marks)

If $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$, find a.

Solution:

$$
\begin{array}{r|l}
 & 2x+5 \\
\hline
x+a & 2x^2+2ax+5x+10 \\
 & 2x^2+2ax \\
 & -\quad - \\
\hline
 & \qquad\quad 5x+10 \\
 & \qquad\quad 5x+5a \\
 & \qquad\ -\quad - \\
\hline
 & \qquad\qquad 10-5a
\end{array}
$$

If $x + a$ is a factor of $2x^2 + 2ax + 5x + 10$, then the remainder i.e., $10 - 5a = 0$ □ $a = 2$.

Question 3 (1.0 marks)

Show that $x = -3$ is a solution of $x^2 + 6x + 9 = 0$.

Solution:

L.H.S = $x^2 + 6x + 9 = (-3)^2 + 6(-3) + 9$ ($\because$ $x = 3$)

$= 9 - 18 + 9$

$= 0 =$ R.H.S

This proves that $x = -3$ is a solution of $x^2 + 6x + 9 = 0$.

Question 4 (1.0 marks)

The first term of an A.P. is p and its common difference is q. Find its 10^{th} term.

Solution:

We know that the nth term of an A.P with the first term a and common difference d is given by:

$$
\begin{aligned}
a_n &= a + (n-1)d \\
\therefore a_{10} &= p + (10-1)q \qquad\qquad (\because a = p,\ d = q,\ n = 10) \\
&= p + 9q
\end{aligned}
$$

Question 5 (1.0 marks)

If $\tan A = \frac{5}{12}$, find the value of $(\sin A + \cos A) \sec A$.

Solution:

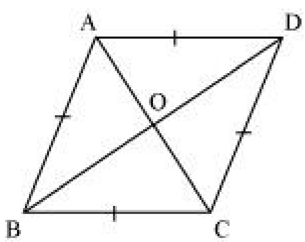
$$(\sin A + \cos A) \sec A = (\sin A + \cos A) \cdot \frac{1}{\cos A}$$
$$= \frac{\sin A}{\cos A} + 1$$
$$= \tan A + 1$$
$$= \frac{5}{12} + 1 = \frac{17}{12}$$

Question 6 (1.0 marks)

The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.

Solution:

Consider a rhombus ABCD.



We know that the diagonals of a rhombus bisect each other at right angles.

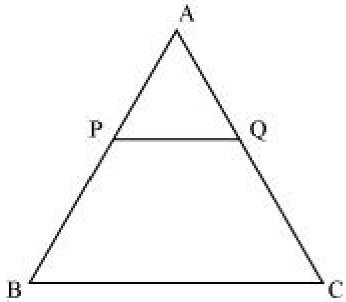
Now, using the Pythagoras theorem for ΔBOC, we get

$$BC^2 = OB^2 + OC^2 = \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 = \left[(20)^2 + (15)^2\right] \text{ cm}^2$$
$$= 625 \text{ cm}^2$$

$\therefore$ Side of rhombus = BC = 25 cm

Question 7 (1.0 marks)

In the figure given below, PQ || BC and AP: PB = 1: 2. Find $\frac{ar(\Delta APQ)}{ar(\Delta ABC)}$.



Solution:

Since AP: PB = 1: 2,

$$\frac{AP}{PB} = \frac{1}{2} \Rightarrow PB = 2AP$$

In ΔAPQ and ΔABC,

□APQ = □ABC (PQ||BC)

□PAQ = □BAC (Common angle)

□ ΔAPQ □ ΔABC (AA similarity)

$$\Rightarrow \frac{\text{area}(\Delta APQ)}{\text{area}(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{AP^2}{(AP+PB)^2} = \frac{AP^2}{(3AP)^2} = \frac{1}{9}$$

Question 8 (1.0 marks)

The surface area of a sphere is 616 cm^2. Find its radius.

Solution:

Surface area of sphere = $4\pi r^2$ = 616 cm^2

$$\Rightarrow r^2 = \left(\frac{616 \times 7}{4 \times 22}\right) cm^2 = 49\, cm^2$$

□ Radius = r = 7 cm

Question 9 (1.0 marks)

A dice is thrown once. Find the probability of getting a number less than 3.

Solution:

When a dice is thrown once, then the sample space = {1, 2, 3, 4, 5, 6}

This means that there are 6 observations in the sample space.

∴ Observations less than 3 = {1, 2}

Thus, there are 2 observations which can give us a number less than 3.

P (number less than 3) = $\frac{2}{6}=\frac{1}{3}$

Question 10 (1.0 marks)

Find the class marks of classes 10 – 25 and 35 – 55.

Solution:

Class mark = $\frac{\text{Lower limit} + \text{Upper limit}}{2}$

Class marks of 10 – 25 = $\frac{10+25}{2}=17.5$

And, class marks of 35 – 55 = $\frac{35+55}{2}=45$

Section B

Question number 11 to 15 carry 2 marks each.

Question 11 (2.0 marks)

Find all the zeroes of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeros are 2 and –2.

Solution:

Since 2 and – 2 are the two zeroes of $x^4 + x^3 - 34x^2 - 4x + 120$, then $(x - 2)(x + 2) = x^2 - 4$ is a factor of $x^4 + x^3 - 34x^2 - 4x + 120$.

$$
\begin{array}{r}
x^2 + x - 30 \\
x^2 - 4 \overline{\big)\; x^4 + x^3 - 34x^2 - 4x + 120} \\
x^4 \qquad - 4x^2 \qquad\qquad\quad \\
\underline{- \qquad + \qquad\qquad\qquad\quad} \\
x^3 - 30x^2 - 4x + 120 \\
x^3 \qquad\quad - 4x \qquad\quad \\
\underline{- \qquad\quad + \qquad\qquad\quad} \\
-30x^2 + 120 \\
-30x^2 + 120 \\
\underline{+ \qquad - \qquad} \\
0
\end{array}
$$

$\therefore x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)\ (x^2 + x - 30)$

$= (x - 2)\ (x + 2)\ (x^2 + 6x - 5x - 30)$

$= (x - 2)\ (x + 2)\ (x + 6)\ (x - 5)$

Hence, the zeroes are 2, – 2, – 6, and 5.

Question 12 (2.0 marks)

A pair of dice is thrown once. Find the probability of getting the same number on each dice.

Solution:

When a pair of dice is thrown, 36 observations are obtained.

There are 6 observations when we get the same number on each dice i.e., (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), and (6, 6).

□ P (same number on each dice) $= \frac{6}{36} = \frac{1}{6}$

Question 13 (2.0 marks)

If sec 4A = cosec (A – 20°) where 4A is an acute angle, then find the value of A.

OR

In Δ ABC right-angled at C, if $\tan A = \frac{1}{\sqrt{3}}$, then find the value of sin A cos B + cos A sin B.

Solution:

$$\sec 4A = \operatorname{cosec}(A - 20°)$$
$$= \sec[90° - (A - 20°)]$$
$$\therefore 4A = 90° - (A - 20°) = 110° - A$$
$$\Rightarrow A = 22°$$

OR,

$$\tan A = \frac{1}{\sqrt{3}} = \tan 30°$$
$$\Rightarrow \angle A = 30°$$

Since ∠C = 90°,

∠A + ∠B = 90°

∴ ∠B = 60°

Now,

$$\sin A \cos B + \cos A \sin B = \sin 30° \cdot \cos 60° + \cos 30° . \sin 60°$$
$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$$
$$= \frac{1}{4} + \frac{3}{4}$$
$$= 1$$

Question 14 (2.0 marks)

Find the value of *k* if the points (*k*, 3), (6, −2), and (−3, 4) are collinear.

Solution:

The points (*k*, 3), (6, − 2), and (− 3, 4) are collinear, if the area of the triangle formed through these points is zero.

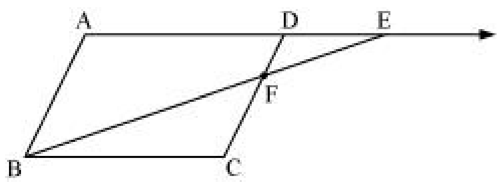
i.e.,

$$\frac{1}{2}\left|k(-2-4)+6(4-3)-3(3+2)\right|=0$$
$$\Rightarrow -6k-9=0$$
$$\Rightarrow k=\frac{-3}{2}$$

Question 15 (2.0 marks)

E is a point on AD produced of a ||gm ABCD and BE intersects CD at F. Show that Δ ABE Δ CFB.

Solution:



In ΔABE and ΔCFB

BAE = FCB (Opposite angles of ||gm ABCD)

AEB = CBF (Alternate interior angles)

 ΔABE ΔCFB (AA similarity)

Section C

Question number 16 to 25 carry 3 marks each.

Question 16 (3.0 marks)

Use Euclid’s Division Lemma to show that the square of any positive integer is either of the form $3m$ or $(3m + 1)$ for some integer m.

Solution:

Let a be any positive integer and $b = 3$.

Then,

$a = 3q + r$ (For some integers $q > 0$)

And,

$r = 0, 1, 2$ (Since $0 \leq r < 3$)

$\therefore a = 3q$ or $3q + 1$ or $3q + 2$

Now,

$a^2 = (3q)^2$ or $(3q + 1)^2$ or $(3q + 2)^2$

 $a^2 = 9q^2$ or $9q^2 + 6q + 1$ or $9q^2 + 12q + 4$

 $a^2 = 3(3q^2)$ or $3(3q^2 + 2q) + 1$ or $3(3q^2 + 4q + 1) + 1$

 $a^2 = 3k_1$ or $3k_2 + 1$ or $3k_3 + 1$, where k_1, k_2, and k_3 are some positive integers.

Hence, it has been shown that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Question 17 (3.0 marks)

Represent the following pair of equations graphically and write the co-ordinates of points where the lines intersect the y-axis:

$x + 3y = 6$

$2x - 3y = 12$

Solution:

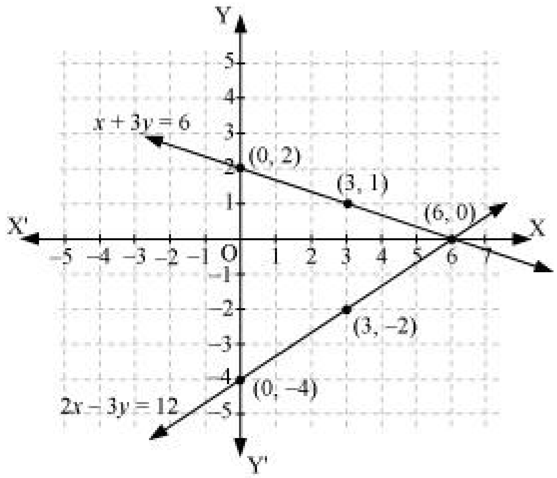
For the linear equation $x + 3y = 6$ i.e., $x = 6 - 3y$, the solution table is as follows:

x	6	3	0
y	0	1	2

For the linear equation $2x - 3y = 12$ i.e., $x = \frac{12 + 3y}{2}$, the solution table is as follows:

X	6	3	0
y	0	−2	−4

The graphical representation of the given linear equations is as follows:



Hence, the co-ordinates of points where the line, $x + 3y = 6$ and the line, $2x - 3y = 12$ intersect the y-axis are (0, 2) and (0, − 4) respectively.

Question 18 (3.0 marks)

For what value of n are the n^{th} terms of two A.P.'s 63, 65, 67, … and 3, 10, 17, … equal?

OR

If m times the m^{th} term of an A.P. is equal to n times its n^{th} term, then find the $(m + n)^{th}$ term of the A.P.

Solution:

For A.P, 63, 65, 67, …

$a = 63$, $d = 65 - 63 = 2$

 $a_n = a + (n - 1)\, d = 63 + (n - 1)\, 2 = 2n + 61$

For AP, 3, 10, 17, ….

$a = 3$, $d = 10 - 3 = 7$

 $a_n = a + (n - 1)\, d = 3 + (n - 1)7 = 7n - 4$

Since the nth terms of the two A.P.'s are equal,

$2n + 61 = 7n - 4$

□ $5n = 65$

□ $n = 13$

OR,

Let the first term and common difference of A.P.'s be a and d respectively.

Now,

$m.\ a_m = n.\ a_n$

$$\Rightarrow m \cdot \left[a + (m-1)d\right] = n \cdot \left[a + (n-1)d\right]$$
$$\Rightarrow (m-n)a + \left(m^2 - m - n^2 + n\right)d = 0$$
$$\Rightarrow (m-n)a + \left[(m-n)(m+n) - (m-n)\right]d = 0$$
$$\Rightarrow (m-n)\left[a + (m+n-1)d\right] = 0$$
$$\Rightarrow a + (m+n-1)d = 0$$
$$\Rightarrow a_{m+n} = 0$$

Question 19 (3.0 marks)

In an A.P. the first term is 8, the n^{th} term is 33, and sum of the first n terms is 123. Find n and d, the common difference.

Solution:

In the given A.P,

$a = 8$

And,

$$a_n = a + (n-1)d = 33$$
$$\Rightarrow 8 + (n-1)d = 33$$
$$\therefore (n-1)d = 25 \qquad \text{...(i)}$$

It is also given that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = 123$$
$$\Rightarrow \frac{n}{2}\left[2 \times 8 + 25\right] = 123$$
$$\Rightarrow n = 6$$

Substituting $n = 6$ in equation (i), we obtain

$5d = 25$ □ $d = 5$

□ $d = 5$ and $n = 6$

Question 20 (3.0 marks)

Prove that:

(1+ cot A + tan A) (sin A − cos A) = sin A tan A − cot A cos A

OR

Without using trigonometric tables, evaluate the following:

$$2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right)-\sqrt{3}\left(\frac{\cos 38^\circ\ \text{cosec} 52^\circ}{\tan 15^\circ\ \tan 60^\circ\ \tan 75^\circ}\right)$$

Solution:

L.H.S = (1 + cot A + tan A) (sin A − cos A)

$$=\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)$$
$$=\left(\frac{\sin A\cos A+\cos^2 A+\sin^2 A}{\sin A\cos A}\right)(\sin A-\cos A)$$
$$=\frac{\sin^3 A-\cos^3 A}{\sin A\cos A} \qquad \left\{a^3-b^3=(a-b)\left(a^2+b^2+ab\right)\right\}$$
$$=\frac{\sin^2 A}{\cos A}-\frac{\cos^2 A}{\sin A}$$
$$=\sin A\tan A-\cos A\cot A$$
$$=\text{R.H.S}$$

OR,

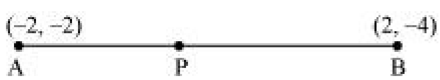
$$2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right)-\sqrt{3}\left(\frac{\cos 38^\circ\cdot\operatorname{cosec} 52^\circ}{\tan 15^\circ\cdot\tan 60^\circ\cdot\tan 75^\circ}\right)$$
$$=2\left[\frac{\cos(90^\circ-32^\circ)}{\sin 32^\circ}\right]-\sqrt{3}\left[\frac{\cos 38^\circ\cdot\operatorname{cosec}(90^\circ-38^\circ)}{\tan(90^\circ-75^\circ)\cdot\sqrt{3}\cdot\tan 75^\circ}\right]$$
$$=2\left(\frac{\sin 32^\circ}{\sin 32^\circ}\right)-\frac{\cos 38^\circ\cdot\sec 38^\circ}{\cot 75^\circ\cdot\tan 75^\circ}$$
$$=2\times 1-\frac{1}{1}$$
$$=1$$

Question 21 (3.0 marks)

If P divides the join of A(−2, −2) and B(2, −4) such that $\frac{AP}{AB}=\frac{3}{7}$, find the co-ordinates of P.

Solution:

$$\because \frac{AP}{AB}=\frac{3}{7}$$
$$\Rightarrow \frac{AP}{PB}=\frac{3}{4}$$



Now, the co-ordinates of P can be given as:

$$\left(\frac{3\times 2+4\times(-2)}{3+4},\frac{3\times(-4)+4\times(-2)}{3+4}\right)$$
$$=\left(\frac{-2}{7},\ \frac{-20}{7}\right)$$

Question 22 (3.0 marks)

The mid-points of the sides of a triangle are (3, 4), (4, 6), and (5, 7). Find the co-ordinates of the vertices of the triangle.

Solution:

Let D(3, 4), E(4, 6), and F(5, 7) be the mid-points of sides AB, BC, and AC of ΔABC (respectively) formed by vertices $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$.

It is given that D is the mid-point of AB.

$$\therefore \frac{x_1 + x_2}{2} = 3 \text{ and } \frac{y_1 + y_2}{2} = 4$$
$$\Rightarrow x_1 + x_2 = 6 \qquad \text{...(i)}$$
And,
$$y_1 + y_2 = 8 \qquad \text{...(ii)}$$

Similarly,

$$\frac{x_2 + x_3}{2} = 4, \ \frac{y_2 + y_3}{2} = 6, \ \frac{x_1 + x_3}{2} = 5, \ \frac{y_1 + y_3}{2} = 7$$
$$\Rightarrow x_2 + x_3 = 8 \qquad \text{...(iii)}$$
$$\Rightarrow y_2 + y_3 = 12 \qquad \text{...(iv)}$$
$$\Rightarrow x_1 + x_3 = 10 \qquad \text{...(v)}$$
$$\Rightarrow y_1 + y_3 = 14 \qquad \text{...(vi)}$$

Adding equations (i), (iii), and (v), we get:

$$2(x_1 + x_2 + x_3) = 24$$
$$\Rightarrow x_1 + x_2 + x_3 = 12 \qquad \text{...(vii)}$$

Similarly, adding equations (ii), (iv), and (vi), we obtain:

$$2(y_1 + y_2 + y_3) = 34$$
$$\Rightarrow y_1 + y_2 + y_3 = 17 \qquad \text{...(viii)}$$

Solving equations (i) and (vii), we get:

$x_3 = 6$

Solving equations (iii) and (vii), we get:

$x_1 = 4$

Solving equations (v) and (viii), we get:

$x_2 = 2$

Similarly, solving equations (ii), (iv), and (vi) with (viii), we get:

$y_3 = 9,\ y_1 = 5,\ y_2 = 3$

Hence, the co-ordinates of the vertices of the triangle are (4, 5), (2, 3), and (6, 9).

Question 23 (3.0 marks)

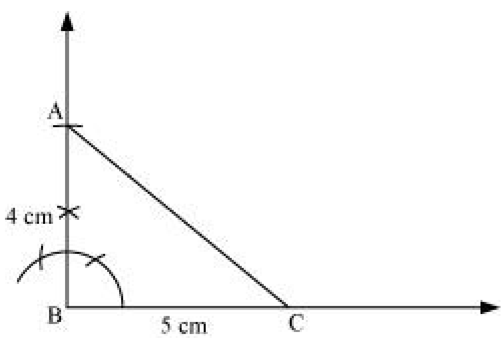
Draw a right triangle where the sides containing the right angle are 5 cm and 4 cm.

Construct a similar triangle whose sides are $\frac{5}{3}$ times the sides of the above triangle.

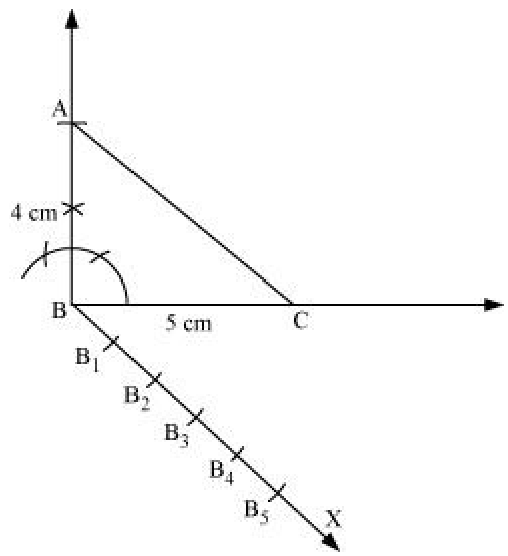
Solution:

Steps of construction:

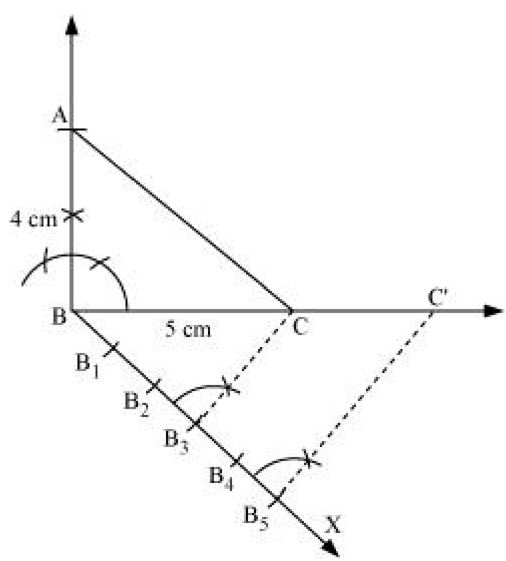
1. Draw a right triangle ABC right-angled at B such that AB = 4 cm and BC = 5 cm.



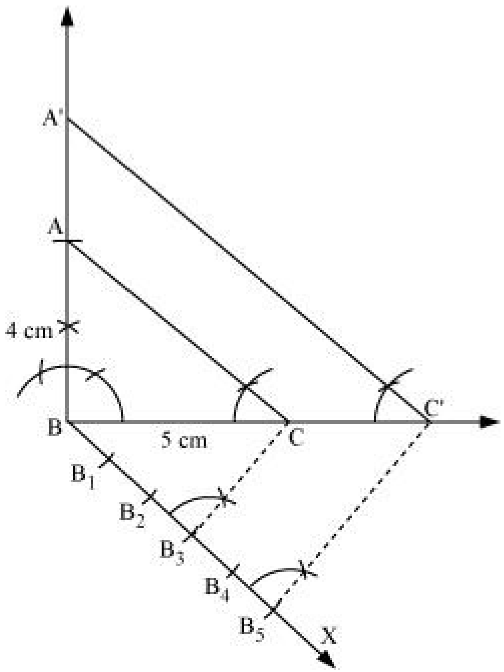
2. Draw a ray BX making an acute angle with BC on the side opposite to vertex A. On BX, locate 5 points B_1, B_2, B_3, B_4, and B_5 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.



3. Join B_3C. Draw a line through B_5 parallel to B_3C, intersecting ray BC at C'.



4. Draw a line through C' parallel to CA, intersecting ray BA at A'.



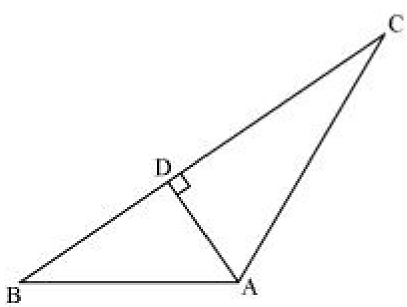
Hence, $\Delta A'BC' \sim \Delta ABC$ where the sides of $\Delta A'BC'$ are $\frac{5}{3}$ times the sides of the right-angled triangle ABC.

Question 24 (3.0 marks)

Prove that a parallelogram circumscribing a circle is a rhombus.

OR

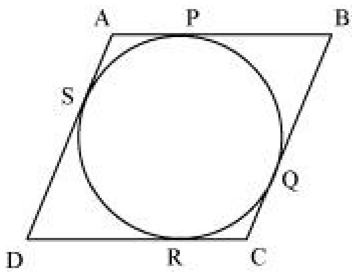
In Figure 2, AD □ BC. Prove that $AB^2 + CD^2 = BD^2 + AC^2$.



Solution:

Consider that $\|^{gm}$ ABCD is circumscribed in a circle.

Sides AB, BC, CD, and AD touch the circle at P, Q, R, and S respectively.



It is known that the lengths of the tangents drawn from an external point to a circle are equal.

$\Rightarrow$ AP = AS … (i)

PB = BQ … (ii)

DR = DS … (iii)

CR = CQ … (iv)

Adding equations (i) to (iv), we obtain

(AP + PB) + (DR + RC) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ 2AB = 2BC (∵ ABCD is a $||^{gm,}$ AB = CD and AD = BC)

$\Rightarrow$ AB = BC

Hence, ABCD is a rhombus since ABCD is a $||^{gm}$ and the adjacent sides are equal (AB = BC).

OR,

Using the Pythagoras theorem for ΔABD, we obtain

$AD^2 + BD^2 = AB^2$

$\Rightarrow$ $AD^2 = AB^2 - BD^2$ … (i)

Using the Pythagoras theorem for ΔACD, we obtain

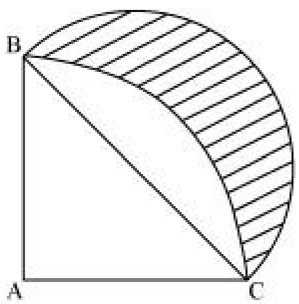
$AD^2 + CD^2 = AC^2$

$\Rightarrow$ $AB^2 - BD^2 + CD^2 = AC^2$ [Using equation (i)]

$\therefore AB^2 + CD^2 = AC^2 + BD^2$

Question 25 (3.0 marks)

In Figure 3, ABC is the quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as the diameter. Find the area of the shaded region.



Solution:

Since ABC is a quadrant of the circle of radius 14 cm,

AB = AC = 14 cm

And,

$\angle BAC = 90°$

$\therefore$Using the Pythagoras theorem for ΔBAC, we obtain

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{(14cm)^2 + (14cm)^2} = 14\sqrt{2} \text{ cm}$$

Now,

Area of shaded region = Area of semi-circle taking BC as diameter

– (Area of quadrant ABC – Area of ΔABC)

$$= \left[\frac{1}{2}\pi \times \left(7\sqrt{2}\right)^2 - \left\{\frac{1}{4} \times \pi \times (14)^2 - \frac{1}{2} \times 14 \times 14\right\}\right]$$
$$= \left[154 - (154 - 98)\right] \text{cm}^2$$
$$= 98 \text{ cm}^2$$

Section D

Question number 26 to 30 carry 6 marks each.

Question 26 (6.0 marks)

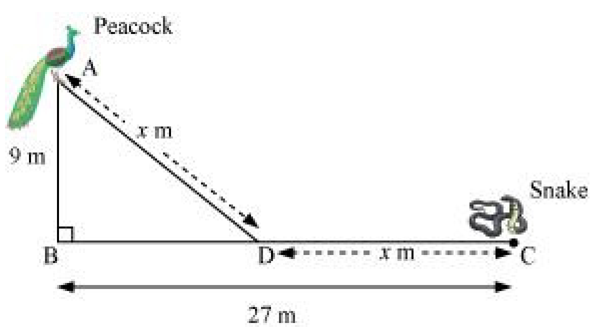
A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?

OR

The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the two numbers.

Solution:

Since the speed of the peacock and snake is equal, they will cover an equal distance in the same time limit. Then, let each animal cover *x* m. The situation can be represented with the help of a figure as below:



Here, AD and DC represent the distance covered by the peacock and the snake respectively.

Using the Pythagoras theorem for right triangle ABD, we obtain:

$AB^2 + BD^2 = AD^2$

$$\Rightarrow (9)^2 + (27 - x)^2 = x^2$$
$$\Rightarrow 81 + 729 + x^2 - 54x = x^2$$
$$\Rightarrow 54x = 810$$
$$\Rightarrow x = 15$$

Now, CD = x m = 15 m

And,

BD = $(27 - x)$ m = (27 − 15) m = 12 m

Hence, the snake is caught at a distance of 12 m from the hole.

OR,

Let the smaller number be x and the larger number be $x + 4$.

It is given that,

$$\frac{1}{x} - \frac{1}{x+4} = \frac{4}{21}$$
$$\Rightarrow \frac{4}{x(x+4)} = \frac{4}{21}$$
$$\Rightarrow x^2 + 4x - 21 = 0$$
$$\Rightarrow x^2 + 7x - 3x - 21 = 0$$
$$\Rightarrow (x+7)(x-3) = 0$$
$$\Rightarrow x = 3 \text{ or } -7$$

If $x = 3$, then $x + 4 = 7$.

If $x = -7$, then $x + 4 = -3$.

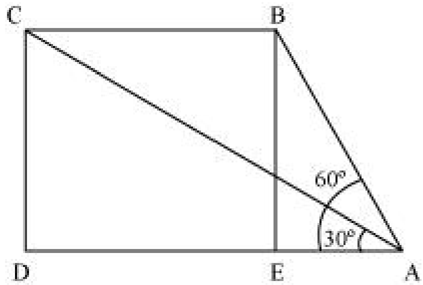
Hence, the numbers are 3 and 7 or − 7 and − 3.

Question 27 (6.0 marks)

The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed in km/hour of the plane.

Solution:

Let B and C be the initial and final positions of the aeroplane respectively.



Height of the aeroplane from the ground, BE = CD = $3600\sqrt{3}$ m

In ΔAEB,

$$\frac{BE}{AE} = \tan 60°$$

$$\Rightarrow \frac{3600\sqrt{3}\text{ m}}{AE} = \sqrt{3}$$

$$\Rightarrow AE = 3600\text{ m}$$

In ΔACD,

$$\frac{CD}{AD} = \tan 30°$$

$$\Rightarrow \frac{3600\sqrt{3}\text{ m}}{AD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AD = 10800\text{ m}$$

Distance travelled by the aeroplane in 30 s = BC

= AD − AE

= (10800 − 3600) m

= 7200 m

 Speed of the aeroplane = $\frac{\text{Distance}}{\text{Time}}$

$$= \frac{7200 \text{ m}}{30 \text{ s}}$$

$$= \frac{\left(\frac{7200}{1000}\right)\text{km}}{\left(\frac{30}{60 \times 60}\right)\text{hr}}$$

$$= \left(\frac{72 \times 120}{10}\right) \text{ km/hr}$$

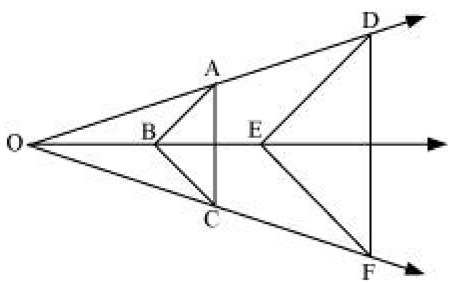
$$= 864 \text{ km/hr}$$

Question 28 (6.0 marks)

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Using the above, prove the following:

In Figure 4, AB || DE and BC || EF. Prove that AC || DF.

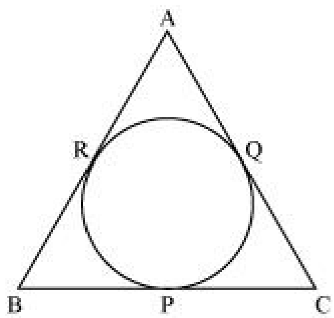


OR

Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Using the above, prove the following:

ABC is an isosceles triangle in which AB = AC circumscribed about a circle, as shown in Figure 5. Prove that the base is bisected by the point of contact.



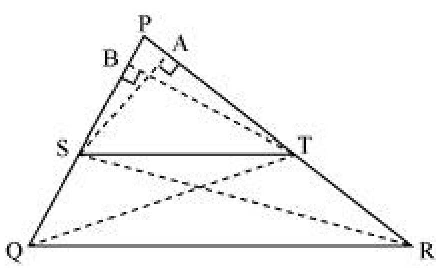
Solution:

Part I:

Let us consider a ΔPQR in which ST ∥ QR, where ST intersects PQ and PR at S and T respectively.

We can prove the given statement, if we can prove $\frac{PS}{QS} = \frac{PT}{RT}$.

For this, let us join RS and QT and draw SA □ PR and TB □ PQ.



$$\text{Now, } \frac{\text{area}(\Delta PST)}{\text{area}(\Delta SQT)} = \frac{\frac{1}{2} \times PS \times BT}{\frac{1}{2} \times QS \times BT} = \frac{PS}{QS} \qquad \text{...(i)}$$

$$\left(\because \text{Area of a triangle} = \frac{1}{2} \text{base} \times \text{altitude} \right)$$

$$\text{Similarly, } \frac{\text{area}(\Delta PST)}{\text{area}(\Delta SRT)} = \frac{\frac{1}{2} \cdot PT \cdot AS}{\frac{1}{2} RT \cdot AS} = \frac{PT}{RT} \qquad \text{...(ii)}$$

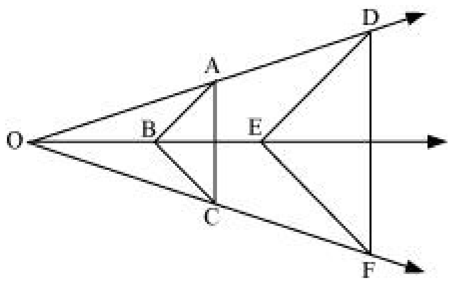
ΔSQT and ΔSRT are on the same base (ST) and between the same parallels (ST and QR).

□ Area (ΔSQT) = Area (ΔSRT) …(iii)

On comparing equations (i), (ii), and (iii), we obtain:

$$\frac{PS}{QS} = \frac{PT}{RT}$$

Part II:



In ΔOED, $AB \parallel DE$

$$\Rightarrow \frac{OA}{AD} = \frac{OB}{BE} \qquad \text{...(iv)}$$

In ΔOEF, $BC \parallel EF$

$$\Rightarrow \frac{OB}{BE} = \frac{OC}{CF} \qquad \text{...(v)}$$

Comparing equations (iv) and (v), we get

$$\frac{OA}{AD} = \frac{OC}{CF}$$

□ $AC \parallel DF$ (converse of B.P.T)

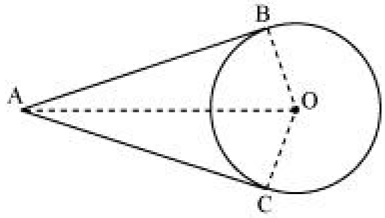
OR,

Part I:

Let us consider a circle with centre O. Let point A be lying outside the circle. Let AB and AC be the tangents drawn to the circle from point A.

We can prove the given statement, if we can prove AB = AC.

For this, let us join OA, OB, and OC.



In ΔOAB and ΔOAC,

OA = OA (Common)

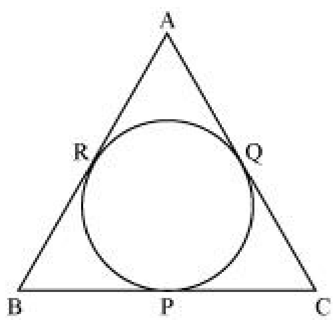
OB = OC (Radii of the same circle)

∠OBA = ∠OCA = 90º (Radius ⊥ tangent)

∴ ΔOAB ≅ ΔOAC (R.H.S congruence)

∴ AB = AC (CPCT)

Part II:



In the given figure, A is a point lying outside the circle. Let AR and AQ be the tangents drawn to the circle.

∴ AR = AQ …(i)

Similarly, BP = BR …(ii)

And, CP = CQ …(iii)

AB = AC …(iv) (Given)

Subtracting equation (i) from equation (iv), we obtain:

AB − AR = AC − AQ

□ BR = CQ

□ BP = CP [Using equations (ii) and (iii)]

Question 29 (6.0 marks)

If the radii of the circular ends of a conical bucket, which is 16 cm high, are 20 cm and 8 cm, find the capacity and the total surface area of the bucket. [Use $\pi = \frac{22}{7}$]

Solution:

The radii of the circular ends of the conical bucket (having the shape of a frustum of a cone) are:

$r_1 = 20 \text{ cm}, r_2 = 8 \text{ cm}$

Height, h = 16 cm

Now,

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$
$$= \left[\sqrt{(16)^2 + (20-8)^2}\right] \text{ cm}$$
$$= 20 \text{ cm}$$

Capacity of the bucket = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times 16\left(20^2 + 8^2 + 20 \times 8\right)\right] \text{ cm}^3$$
$$= \frac{73216}{7} \text{ cm}^3$$
$$= 10459 \cdot 43 \text{ cm}^3$$

Total surface area of the bucket (circular base of radius 20 cm is open):

$= \pi\left[(r_1 + r_2)l + r_2^2\right]$

$= \frac{22}{7}\left[(20+8)\times 20 + (8)^2\right] \text{ cm}^2$

$= \left(\frac{13728}{7}\right) \text{ cm}^2$

$= 1961.14 \text{ cm}^2$

Question 30 (6.0 marks)

Find mean, median, and mode of the following data:

Classes	Frequency
0 – 20	6
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	6
100 – 120	5
120 – 140	3

Solution:

Class interval	**Frequency (f_i)**	**Class marks x_i**	$f_i x_i$
0 – 20	6	10	60
20 – 40	8	30	240
40 – 60	10	50	500
60 – 80	12	70	840
80 – 100	6	90	540
100 – 120	5	110	550
120 – 140	3	130	390
	$\sum f_i = 50$		$\sum f_i x_i = 3120$

Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{3120}{50} = 62.4$

Class interval 60 – 80 has maximum frequency. Now, the modal class is 60 – 80.

 l = lower limit of the modal class = 60

h = size of the modal class = 80 – 60 = 20

f_1 = frequency of the modal class = 12

f_0 = frequency of the class preceding the modal class = 10

f_2 = frequency of the class succeeding the modal class = 6

$$\therefore \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$
$$= 60 + \left(\frac{12 - 10}{24 - 10 - 6} \right) \times 20$$
$$= 60 + 5$$
$$= 65$$

To find the median, we have to calculate the cumulative frequency of each class interval as shown below:

Class interval	***f***	***cf***
0 – 20	6	6
20 – 40	8	14
40 – 60	10	24
60 – 80	12	36
80 – 100	6	42
100 – 120	5	47
120 – 140	3	50
	n = 50	

Now, $\frac{n}{2}=\frac{50}{2}=25$

The 25th observation lies in class 60 – 80.

$\therefore$ Median class = 60 – 80

 l = lower limit of the median class = 60

cf = cumulative frequency of the class preceding the median class = 24

f = frequency of the median class = 12

h = class size = 20

$$\therefore \text{Median} = l+\left[\frac{\frac{n}{2}-cf}{f}\right]\times h$$
$$=60+\left(\frac{25-24}{12}\right)\times 20$$
$$=60+\frac{5}{3}$$
$$=61.67$$

Question 8 (1.0 marks)

Show that $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$.

Solution:

The given equation is $3x^2 + 13x + 14 = 0$.

For $x = -2$,

L.H.S $= 3(-2)^2 + 13(-2) + 14$

$= 3(4) + 13(-2) + 14$

$= 12 - 26 + 14$

$= 26 - 26 = 0 =$ R.H.S

Hence, $x = -2$ is a solution of the equation $3x^2 + 13x + 14 = 0$.

Question 10 (1.0 marks)

A dice is thrown once. Find the probability of getting a number greater than 5.

Solution:

$$\text{Probability} = \frac{\text{Number of possible outcomes of the event}}{\text{Total number of outcomes}}$$

When a dice is thrown once, the possible outcomes are 1, 2, 3, 4, 5, and 6. Out of these, the number greater than 5 is 6.

Thus, the probability of getting a number greater than 5 is the same as the probability of obtaining the number 6.

$\Rightarrow$ Required probability $= \frac{1}{6}$

Question 14 (2.0 marks)

Find all the zeroes of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

Solution:

The given polynomial is $p(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30.$

It is given that $\sqrt{2}$ and $-\sqrt{2}$ are the zeroes of the polynomial.

$\therefore \left(x-\sqrt{2}\right)$ and $\left(x+\sqrt{2}\right)$ are the factors of the given polynomial $p(x)$.

i.e., $(x^2 - 2)$ is a factor of polynomial $p(x)$.

Now, we divide $p(x)$ by $(x^2 - 2)$:

$$
\begin{array}{r|l}
 & 2x^2 + 7x - 15 \\
x^2 - 2 & 2x^4 + 7x^3 - 19x^2 - 14x + 30 \\
 & 2x^4 \qquad\quad - 4x^2 \\
 & - \qquad\qquad + \\
\hline
 & \qquad 7x^3 - 15x^2 - 14x + 30 \\
 & \qquad 7x^3 \qquad\quad - 14x \\
 & \qquad - \qquad\qquad + \\
\hline
 & \qquad\qquad -15x^2 \qquad + 30 \\
 & \qquad\qquad -15x^2 \qquad + 30 \\
 & \qquad\qquad + \qquad\qquad - \\
\hline
 & \qquad\qquad\qquad 0 \\
\hline
\end{array}
$$

$\therefore p(x) = (x^2 - 2)(2x^2 + 7x - 15)$

Now, $2x^2 + 7x - 15 = (2x - 3)(x + 5)$

$\therefore p(x) = \left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)(2x-3)(x+5)$

Hence, the zeroes of the given polynomial are $\sqrt{2}, -\sqrt{2}, \frac{3}{2}$, and -5.

Question 15 (2.0 marks)

A bag contains tickets numbered 11, 12, 13, …, 30. A ticket is taken out from the bag at random. Find the probability that the number on the drawn ticket (i) is a multiple of 7, (ii) is greater than 15 a multiple of 5.

Solution:

It is given that a bag contains tickets numbered 11, 12, …, 30.

A ticket is drawn from the bag at random.

Total outcomes = 20

(i) Out of these numbers, the multiples of 7 are 14, 21, and 28.

We know that probability of an event is given by,

$$\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

 Probability (multiple of 7) $= \frac{3}{20}$

(ii) Out of these numbers, numbers that are greater than 15 and also a multiple of 5 are 20, 25, and 30.

 p (number greater than 15 and a multiple of 5) $= \frac{3}{20}$

Question 16 (3.0 marks)

In an A.P., the first term is 22, n^{th} term is − 11, and sum to first *n* terms is 66. Find *n* and *d,* the common difference.

Solution:

Let *a* and *d* be the first term and the common difference of the given A.P respectively.

Given,

$a = 22$

$a_n = -11$

$S_n = 66$

We know that the n^{th} term of an A.P. is given by,

$$a_n = a + (n-1)d$$
$$-11 = 22 + (n-1)d$$
$$\Rightarrow (n-1)d = -11 - 22$$
$$\Rightarrow (n-1)d = -33 \qquad \ldots(1)$$

Also, we know that the sum of *n* terms of an A.P. is given by,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$66 = \frac{n}{2}\left[2 \times 22 + (n-1)d\right]$$
$$\Rightarrow 66 = \frac{n}{2}[44 - 33] \qquad [\text{From equation (1)}]$$
$$\Rightarrow 132 = n(11)$$
$$\Rightarrow n = \frac{132}{11} = 12$$

Now, from equation (1), we have:

$$(12-1)d = -33$$
$$\Rightarrow 11d = -33$$
$$\Rightarrow d = -3$$

Hence, the values of *n* and *d* are 12 and − 3 respectively.

Question 24 (3.0 marks)

Find the ratio in which the line $3x + y - 9 = 0$ divides the line-segment joining the points (1, 3) and (2, 7).

Solution:

Let the line $3x + y - 9 = 0$ divide the line segment joining the points A (1, 3) and B (2, 7) in the ratio k:1 at point P(x, y).

Then, by the section formula, we have:

$$x = \frac{2k+1}{k+1} \text{ and } y = \frac{7k+3}{k+1}$$

Also, point P (x, y) will lie on line $3x + y - 9 = 0$.

Thus, we have:

$$3\left(\frac{2k+1}{k+1}\right)+\left(\frac{7k+3}{k+1}\right)-9=0$$
$$\Rightarrow 3(2k+1)+(7k+3)-9(k+1)=0$$
$$\Rightarrow 6k+3+7k+3-9k-9=0$$
$$\Rightarrow 4k-3=0$$
$$\Rightarrow k=\frac{3}{4}$$

Hence, the required ratio is 3:4.

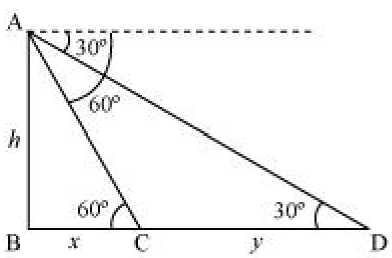
Question 26 (3.0 marks)

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at the angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Solution:

Let AB be the tower and A be the position of the man.

Initially, the car is at point D. Six seconds later, the car is at point C.



Let *h* be the height of tower AB.

Let BC = *x* and CD = *y*.

Now, the distance covered by the car in 6 seconds is CD.

In ΔABC, we have:

$$\tan 60° = \frac{AB}{BC}$$
$$\Rightarrow \sqrt{3} = \frac{h}{x}$$
$$\Rightarrow h = \sqrt{3}x \qquad ...(1)$$

In ΔADB, we have:

$$\tan 30° = \frac{AB}{BD}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y}$$
$$\Rightarrow x + y = \sqrt{3}h$$
$$\Rightarrow x + y = \sqrt{3}\left(\sqrt{3}x\right) \qquad (\text{Using}(1))$$
$$\Rightarrow x + y = 3x$$
$$\Rightarrow 2x = y$$
$$\Rightarrow x = \frac{1}{2}y$$

Now, it is given that the car travels distance CD (= *y*) in 6 seconds.

∴ Time taken by the car to reach the foot of the tower i.e., to cover distance BC = *x* = $\frac{1}{2}y$ i.e., half of 6 seconds = 3 seconds.

Question 7 (1.0 marks)

Show that $x = -3$ is a solution of $2x^2 + 5x - 3 = 0$.

Solution:

The given equation is $2x^2 + 5x - 3 = 0$.

For $x = -3$, we have:

L.H.S = $2(-3)^2 + 5(-3) - 3$

$= 2(9) + 5(-3) - 3$

$= 18 - 15 - 3$

$= 18 - 18 = 0 =$ R.H.S

Hence, $x = -3$ is a solution of the equation $2x^2 + 5x - 3 = 0$.

Question 9 (1.0 marks)

If $\cos A = \frac{7}{25}$, find the value of tan A + cot A.

Solution:

We have $\cos A = \frac{7}{25}$.

We know that $\sin^2 A + \cos^2 A = 1$.

$$\sin^2 A = 1 - \left(\frac{7}{25}\right)^2$$

$$= 1 - \frac{49}{625}$$

$$= \frac{625 - 49}{625} = \frac{576}{625}$$

$$\Rightarrow \sin A = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{24}{25}}{\frac{7}{25}} = \frac{24}{7}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{7}{25}}{\frac{24}{25}} = \frac{7}{24}$$

$$\therefore \tan A + \cot A = \frac{24}{7} + \frac{7}{24}$$

$$= \frac{(24)^2 + (7)^2}{168} = \frac{576 + 49}{168} = \frac{625}{168}$$

Hence, the value of $(\tan A + \cot A)$ is $\frac{625}{168}$.

Question 13 (2.0 marks)

If sec 2A = cosec (A – 42°), where 2A is an acute angle, find the value of A.

OR,

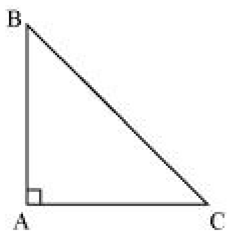
In a ΔABC, right-angled at A, if $\tan C = \sqrt{3}$, find the value of sin B cos C + cos B sin C.

Solution:

We have:

$\sec 2A = \operatorname{cosec}(A - 42°)$
$\Rightarrow \sec 2A = \sec[90° - (A - 42°)]$
$\Rightarrow \sec 2A = \sec(132° - A)$
$\Rightarrow 2A = 132° - A$
$\Rightarrow 3A = 132°$
$\Rightarrow A = \frac{132°}{3} = 44°$

OR,



In ΔABC,

$\angle A = 90°$

It is given that $\tan C = \sqrt{3}.$

$\tan C = \tan 60°$

$\therefore \angle C = 60°$

Also, we now that the sum of all the angles of a triangle is 180°.

$\therefore \angle A + \angle B + \angle C = 180°$

$\Rightarrow 90° + \angle B + 60° = 180°$

$\Rightarrow 150° + \angle B = 180°$

$\Rightarrow \angle B = 180° - 150°$

$\Rightarrow \angle B = 30°$

Thus, we have:

$$\sin B\cos C + \cos B\sin C = \sin 30^\circ\cos 60^\circ + \cos 30^\circ \sin 60^\circ$$
$$= \frac{1}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}$$
$$= \frac{1}{4} + \frac{3}{4}$$
$$= \frac{4}{4} = 1$$

Question 15 (2.0 marks)

A bag contains 4 red, 5 black and 3 yellow balls. A ball is taken out of the bag at random. Find the probability that the ball taken out is of (i) yellow colour (ii) not of red colour.

Solution:

It is given that the bag contains 4 red, 5 black, and 3 yellow balls. A ball is drawn from the bag at random.

Total number of balls = 4 + 5 + 3 = 12

Probability of an event = $\dfrac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

i. Probability (yellow colour ball) = $\dfrac{3}{12} = \dfrac{1}{4}$
ii. Probability (red colour ball) = $\dfrac{4}{12} = \dfrac{1}{3}$

Probability (ball not red in colour) = $1 - \dfrac{1}{3} = \dfrac{2}{3}$

Question 20 (3.0 marks)

Show that A(− 3, 2), B(− 5, − 5), C(2, − 3), and D(4, 4) are the vertices of a rhombus.

Solution:

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is given by,

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The given vertices are A(− 3, 2), B(− 5, − 5), C(2, − 3), and D(4, 4).

We know that in a rhombus, all sides are of equal length. Thus, vertices A, B, C, and D will be the vertices of a rhombus if AB = BC = CD = DA.

Then, we have:

$$\text{AB}=\sqrt{\left[-5-(-3)\right]^2+(-5-2)^2}=\sqrt{(-5+3)^2+(-7)^2}=\sqrt{4+49}=\sqrt{53}$$
$$\text{BC}=\sqrt{\left[2-(-5)\right]^2+\left[-3-(-5)\right]^2}=\sqrt{(2+5)^2+(-3+5)^2}=\sqrt{49+4}=\sqrt{53}$$
$$\text{CD}=\sqrt{(4-2)^2+\left[4-(-3)\right]^2}=\sqrt{(2)^2+(7)^2}=\sqrt{4+49}=\sqrt{53}$$
$$\text{DA}=\sqrt{(-3-4)^2+(2-4)^2}=\sqrt{(-7)^2+(-2)^2}=\sqrt{49+4}=\sqrt{53}$$

Now, we may find that AB = BC = CD = DA.

Hence, ABCD is a rhombus.

Question 25 (3.0 marks)

In an A.P., the first term in 25, *n*th term is − 17 and sum to first *n* terms is 60. Find *n* and *d*, the common difference.

Solution:

Let *a* and *d* be the first term and the common difference of an A.P. respectively.

Given,

$a = 25$

$a_n = -17$

$S_n = 60$

*n*th term of AP is given by,

$$a_n = a+(n-1)d$$
$$-17 = 25+(n-1)d$$
$$\Rightarrow (n-1)d = -17-25 = -42 \qquad \text{...(i)}$$

We also know that the sum of the first *n* terms of an AP is given by,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$60 = \frac{n}{2}\left[2 \times 25 + (n-1)d\right]$$

$$\Rightarrow 60 = \frac{n}{2}[50 - 42] \qquad \left[\text{From equation (i)}\right]$$

$$\Rightarrow 120 = n(8)$$

$$\Rightarrow n = \frac{120}{8}$$

$$\Rightarrow n = 15$$

Placing the value of *n* in (i), we have:

$$(15-1)d = -42$$

$$\Rightarrow d = \frac{-42}{14} = -3$$

Question 26 (6.0 marks)

Find mean, median and mode of the following data:

Classes	**Frequency**
0 – 50	2
50 – 100	3
100 – 150	5
150 – 200	6
200 – 250	5
250 – 300	3
300 – 350	1

Solution:

We can draw a table as follows:

Classes	**Frequency**	**Class marks (x_i)**	**Cumulative frequency (*c.f.*)**	$f_i x_i$

0 – 50	2	25	2	50
50 – 100	3	75	5	225
100 – 150	5	125	10	625
150 – 200	6	175	16	1050
200 – 250	5	225	21	1125
250 – 300	3	275	24	825
300 – 350	1	325	25	325
Total	25			4225

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{4225}{25} = 169$$

Here, the maximum class frequency is 6.

∴ Modal class = 150 – 200

$$\text{Now, Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Here, } l = 150,\ f_1 = 6,\ f_0 = 5,\ f_2 = 5,\ h = 50.$$

$$\therefore \text{Mode} = 150 + \left(\frac{6-5}{12-5-5}\right) \times 50$$

$$= 150 + \frac{1}{2} \times 50 = 150 + 25 = 175$$

$$\text{Now, Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Here, $n = 25$ and the median class is 150 – 200 as the cumulative frequency is just greater than $\frac{n}{2} = 12.5$ i.e.,16, which belongs to the class interval 150 – 200.

$l = 150$, $cf = 10$, $f = 6$, $h = 50$

$$\therefore \text{Median} = 150 + \left(\frac{\frac{25}{2} - 10}{6} \right) \times 50$$

$$= 150 + \frac{5}{2 \times 6} \times 50 = 150 + 20.83 = 170.83$$

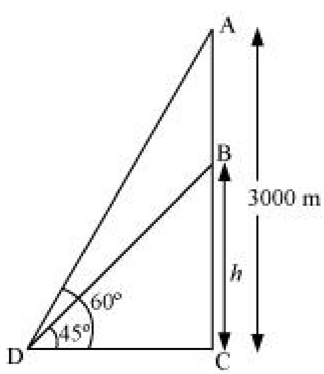
Question 30 (6.0 marks)

An aeroplane, when 3000 m high, passes vertically above another aero plane at an instant, when the angle of elevation of the two aero planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aero planes.

(Use $\sqrt{3} = 1.732$)

Solution:

According to the given information, we can draw the required figure as follows:



Here, points A and B represent the positions of the two aeroplanes.

We need to find the vertical distance between the aeroplanes i.e., length AB.

In ΔADC, we have:

$$\tan 60^\circ = \frac{AC}{DC}$$

$$\Rightarrow \sqrt{3} = \frac{3000}{DC}$$

$$\Rightarrow DC = \frac{3000}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3000\sqrt{3}}{3} = 1000\sqrt{3}$$

$$DC = 1000 \times 1.732$$

$$DC = 1732 \text{ m} \qquad \text{...(i)}$$

Also, in ΔBDC,

$$\tan 45^\circ = \frac{BC}{DC}$$

$$\Rightarrow 1 = \frac{h}{DC}$$

$$\Rightarrow h = DC$$

∴ h = 1732 m [From equation (i)]

 Distance between the two aeroplanes is

AB = AC − BC = 3000 m − h = (3000 − 1732) m = 1268 m.

Hence, the vertical distance between the two aeroplanes is 1268 m.