Question 1 (1.0 marks)

The decimal expansion of the rational number $\frac{43}{2^4.5^3}$, will terminate after how many places of decimals?

Solution:

The given expression i.e., $\frac{43}{2^4.5^3}$ can be rewritten as

$$\frac{43}{2^4.5^3}=\frac{43}{2\times2\times2\times2\times5\times5\times5}=\frac{43}{(2\times5)\times(2\times5)\times(2\times5)\times2}$$

$$=\frac{43}{1000\times2}=\frac{0.043}{2}$$

Now, on dividing 0.043 by 2, we obtain $\frac{0.043}{2}$ = 0.0215.

Thus, the given expression will terminate after 4 decimal places.

Question 2 (1.0 marks)

For what value of k, (–4) is a zero of the polynomial $x^2 - x - (2k + 2)$?

Solution:

Let $p(x) = x^2 - x - (2k + 2)$.

If (–4) is a zero of $p(x)$, then $p(-4) = 0$.

$p(-4) = 0$

Þ $(-4)^2 - (-4) - 2k - 2 = 0$

Þ $16 + 4 - 2k - 2 = 0$

Þ $18 - 2k = 0$

Þ $2k = 18$

Þ $k = 9$

Thus, the required value of k is 9.

Question 3 (1.0 marks)

For what value of p, are $2p - 1$, 7 and $3p$ three consecutives terms of an A.P.?

Solution:

Let $2p - 1$, 7, and $3p$ be the three consecutive terms of an A.P.

The difference between any two consecutive terms of an A.P. is equal.

=>2nd term − 1st term = 3rd term − 2nd term

=> $7 - (2p - 1) = 3p - 7$

=> $7 - 2p + 1 = 3p - 7$

=> $8 - 2p = 3p - 7$

=> $15 = 5p$

$$\Rightarrow p = \frac{15}{5} = 3$$

Thus, if $p = 3$, then $2p - 1$, 7, and $3p$ are the three consecutive terms of an A.P.

Question 4 (1.0 marks)

In Fig. 1, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR.

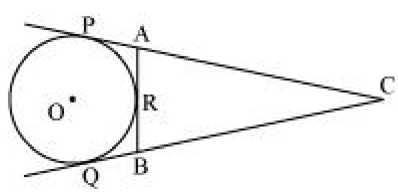


Fig. 1

Solution:

It is given that CP and CQ are the tangents from the same point C.

We know that the lengths of tangents drawn from an external point to a circle are equal.

CP = CQ = 11 cm [CP = 11 cm]

=>CQ = 11 cm

=> CB + BQ = 11 cm

=> 7 cm + BQ = 11 cm [BC = 7 cm]

=> BQ = 11 cm – 7 cm = 4 cm

According to the given figure, BQ and BR are the tangents from the same point B.

=>BR = BQ = 4 cm.

Thus, the length of BR is 4 cm.

Question 5 (1.0 marks)

In Fig. 2, ∠M = ∠N = 46°. Express *x* in terms of *a*, *b* and *c* where *a*, *b* and *c* are lengths, of LM, MN and NK respectively.

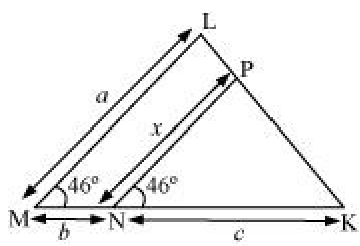


Fig. 2

Solution:

In ΔKPN and ΔKLM, we have

∠KNP = ∠KML = 46° [Given]

∠NKP = ∠MKL [Common]

Thus, ΔKPN ∼ ΔKLM [by AA similarity criterion of triangles]

$$\Rightarrow \frac{KN}{KM} = \frac{PN}{LM} \qquad \left[\text{If two triangles are similar, then their corresponding sides are proportional}\right]$$

$$\Rightarrow \frac{c}{c+b} = \frac{x}{a} \qquad [KM = KN + NM]$$

$$\therefore x = \frac{ac}{b+c}$$

Question 6 (1.0 marks)

If $\sin\theta = \frac{1}{3}$, then find the value of $(2\cot^2\theta + 2)$.

Solution:

It is given that $\sin\theta = \frac{1}{3}$.

$$\therefore \cos\theta = \sqrt{1-\sin^2\theta} \qquad \left[\sin^2\theta + \cos^2\theta = 1\right]$$

$$= \sqrt{1-\left(\frac{1}{3}\right)^2} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\left(\frac{2\sqrt{2}}{3}\right)}{\left(\frac{1}{3}\right)} = 2\sqrt{2}$$

$$\therefore \left(2\cot^2\theta + 2\right) = 2(2\sqrt{2})^2 + 2 = 2\times 8 + 2 = 16 + 2 = 18$$

Question 7 (1.0 marks)

Find the value of a so that the point $(3, a)$ lies on the line represented by $2x - 3y = 5$.

Solution:

If the point $(3, a)$ lies on the line represented by $2x - 3y = 5$, then the point

$(3, a)$ satisfies the equation $2x - 3y = 5$.

$\Rightarrow 2 \times 3 - 3 \times a = 5$

$\Rightarrow 6 - 3a = 5$

$\Rightarrow 6 - 5 = 3a$

$\Rightarrow 1 = 3a$

$$\Rightarrow a = \frac{1}{3}$$

Thus, the required value of a is $\frac{1}{3}$.

Question 8 (1.0 marks)

A cylinder and a cone are of same base radius and of same height. Find the ratio of the volume of cylinder to that of the cone.

Solution:

It is given that the cylinder and the cone are of same base radius and same height.

Therefore, let

Radius of cylinder = Radius of cone = r

Height of cylinder = Height of cone = h

=>Volume of cylinder = $\pi r^2 h$

Volume of cone = $\frac{1}{3}\pi r^2 h$

$$\therefore \frac{\text{Volume of cylinder}\pi}{\text{Volume of cone}} = \frac{r^2 h \ 1}{\left(\frac{1}{3}\pi r^2 h\right)} = \frac{3}{\left(\frac{1}{3}\right)} = \frac{}{1} = 3:1$$

Thus, the ratio of the volume of the cylinder to that of the cone is 3:1.

Question 9 (1.0 marks)

Find the distance between the points $\left(\frac{-8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$.

Solution:

Let the given points be A $\left(\frac{-8}{5}, 2\right)$ and B $\left(\frac{2}{5}, 2\right)$.

We know that the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\therefore AB = \sqrt{\left[\frac{2}{5} - \left(\frac{-8}{5}\right)\right]^2 + (2-2)^2} = \sqrt{\left(\frac{2}{5} + \frac{8}{5}\right)^2 + 0} = \sqrt{\left(\frac{10}{5}\right)^2} = \sqrt{(2)^2} = 2 \text{ units}$$

Thus, the distance between the given points is 2 units.

Question 10 (1.0 marks)

Write the median class of the following distribution:

Classes Frequency

0–10 4

10–20 4

20–30 8

30–40 10

40–50 12

50–60 8

60–70 4

Solution:

The given data can be written in the tabular form as

Class	Frequency	Cumulative frequency
0–10	4	4
10–20	4	8
20–30	8	16
30–40	10	26
40–50	12	38
50–60	8	46
60–70	4	50

Here, $n = 50 \Rightarrow \frac{n}{2} = \frac{50}{2} = 25$

Median class is that class whose cumulative frequency is greater than and nearest to $\frac{n}{2}$.

Thus, the median class of the given distribution is 30–40.

Section B

Question Number 11 to 15 carry 2 marks each.

Question 11 (2.0 marks)

If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find *a* and *b*.

Solution:

The given polynomial i.e., $6x^4 + 8x^3 + 17x^2 + 21x + 7$ can be divided by $3x^2 + 4x + 1$ as

$$\begin{array}{r|l} & 2x^2 + 5 \\ 3x^2 + 4x + 1 & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\ & 6x^4 + 8x^3 + 2x^2 \\ & - \quad - \quad - \\ \hline & 15x^2 + 21x + 7 \\ & 15x^2 + 20x + 5 \\ & - \quad - \quad - \\ \hline & x + 2 \end{array}$$

Thus, the remainder obtained is $x + 2$.

It is given that the remainder is of the form $ax + b$.

On comparing both of them, we obtain $a = 1$ and $b = 2$.

Thus, the respective values of *a* and *b* are 1 and 2.

Question 12 (2.0 marks)

Find the value(s) of k for which the pair of linear equations $kx + 3y = k - 2$ and $12x + ky = k$ has no solution.

Solution:

The given equations are $kx + 3y = k - 2$ and $12x + ky = k$.

The above equations can be rewritten as

$kx + 3y - (k - 2) = 0$, and

$12x + ky - k = 0$

The equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

On comparing the given equations, we obtain

$a_1 = k$, $a_2 = 12$, $b_1 = 3$, $b_2 = k$, $c_1 = -(k - 2)$, $c_2 = -k$

$\therefore \frac{k}{12} = \frac{3}{k} \neq \frac{-(k-2)}{-k}$

On considering $\frac{k}{12} = \frac{3}{k}$, we obtain $k^2 = 36$, which gives $k = \pm 6$

Thus, for $k = 6$ and -6, the given set of equations will have no solutions.

Question 13 (2.0 marks)

If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$, then find its nth term.

Solution:

It is given that $S_n = 3n^2 - 4n$.

We know that $S_n - S_{n-1} = a_n$.

$$\Rightarrow 3n^2 - 4n - \{3(n-1)^2 - 4(n-1)\} = a_n$$
$$\Rightarrow 3n^2 - 4n - \{3(n^2 + 1 - 2n) - 4n + 4\} = a_n$$
$$\Rightarrow 3n^2 - 4n - \{3n^2 + 3 - 6n - 4n + 4\} = a_n$$
$$\Rightarrow 3n^2 - 4n - 3n^2 - 7 + 10n = a_n$$
$$\Rightarrow 6n - 7 = a_n$$

Thus, the n^{th} term of the given A.P. is $(6n - 7)$.

Question 14 (2.0 marks)

Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle$APB = 2 $\angle$OAB.

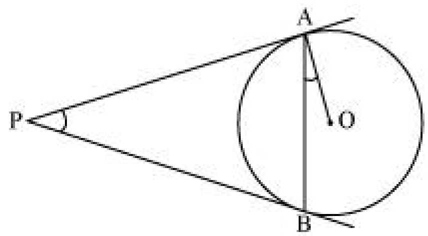
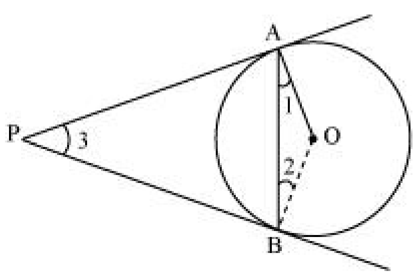


Fig. 3

OR

Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:



We start by joining OB.

We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

=> $\angle$OAP = $\angle$OBP = 90º

Now,

$\angle OAP + \angle 3 + \angle OBP + \angle AOB = 360°$ [Angle sum property of quadrilaterals]

=> $90° + \angle 3 + 90° + \angle AOB = 360°$

=> $\angle AOB = 360° - 180° - \angle 3 = 180° - \angle 3$ (1)

Now, in ΔOAB, OA is equal to OB as both are radii.

=> $\angle 1 = \angle 2$ [In a triangle, angles opposite to equal sides are equal]

Now, on applying angle sum property of triangles in ΔAOB, we obtain

$\angle 1 + \angle 2 + \angle AOB = 180°$

=> $2\angle 1 + \angle AOB = 180°$

=> $2\angle 1 + (180° - \angle 3) = 180°$ [Using (1)]

=> $2\angle 1 = \angle 3$

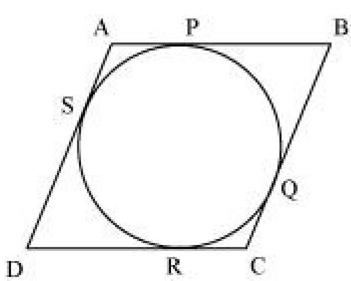
=> $\angle APB = 2\angle OAB$

Thus, the given result is proved.

OR

Consider that parallelogram ABCD circumscribes a circle.

Sides AB, BC, CD, and AD touch the circle at points P, Q, R, and S respectively.



It is known that the lengths of the tangents drawn from an external point to a circle are equal.

=> AP = AS … (i)

PB = BQ … (ii)

DR = DS … (iii)

CR = CQ … (iv)

On adding all the equations, we obtain

(AP + PB) + (DR + RC) = (AS + DS) + (BQ + CQ)

=> AB + CD = AD + BC

=> 2AB = 2BC [∵ ABCD is a $\|^{gm}$, AB = CD and AD = BC]

=> AB = BC

Thus, we can say that ABCD is a rhombus since ABCD is a parallelogram and its adjacent sides are equal (AB = BC).

Question 15 (2.0 marks)

Simplify: $\dfrac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$

Solution:

$$\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$$
$$=\frac{(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)}{\sin\theta+\cos\theta}+\sin\theta\cos\theta \qquad \left[\because a^3+b^3=(a+b)(a^2+b^2-ab)\right]$$
$$=(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)+\sin\theta\cos\theta$$
$$=1-\sin\theta\cos\theta+\sin\theta\cos\theta \qquad \left[\sin^2\theta+\cos^2\theta=1\right]$$
$$=1$$

Thus, the value of the given expression is 1.

Section C

Question Number 16 to 25 carry 3 marks each.

Question 16 (3.0 marks)

Prove that $\sqrt{5}$ is an irrational number.

Solution:

Let us assume on the contrary that $\sqrt{5}$ is rational. Accordingly, there exist positive co-primes *a* and *b* such that

$$\sqrt{5} = \frac{a}{b}$$
$$5 = \frac{a^2}{b^2}$$
$$b^2 \times 5 = a^2$$

5 is a factor of a^2.

This implies that 5 is a factor of a.

$a = 5c$ for some positive integer *c*

Now, $b^2 \times 5 = a^2$

$$b^2 \times 5 = 5^2 \times c^2$$
$$b^2 = 5 \times c^2$$

This means that b^2 is a multiple of 5, or *b* is a multiple of 5.

Therefore, we see that both a and *b* are multiples of 5 or 5 is a factor of both a and *b*.

This contradicts that *a* and *b* are co-primes, which has happened because of our incorrect assumption.

Hence, $\sqrt{5}$ is an irrational number.

Question 17 (3.0 marks)

Solve the following pair of equations:

$$\frac{5}{x-1} + \frac{1}{y-2} = 2$$
$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

Solution:

The given pair of linear equations is

$$\frac{5}{x-1}+\frac{1}{y-2}=2 \quad ...(1)$$

$$\frac{6}{x-1}-\frac{3}{y-2}=1 \quad ...(2)$$

Let $\frac{1}{x-1}=u$ and $\frac{1}{y-2}=v$

Accordingly, equations (1) and (2) become

$5u + v = 2$ … (3)

$6u - 3v = 1$ … (4)

On multiplying equation (3) with 3 and adding it to equation (4), we obtain

$$\begin{array}{l} 15u+3v=6 \\ \underline{\ \ 6u-3v=1} \\ 21u \quad\quad =7 \end{array}$$

$$\Rightarrow u=\frac{1}{3}$$

On substituting the value of u in equation (3), we obtain

$$5\left(\frac{1}{3}\right)+v=2$$

$$\Rightarrow \frac{5}{3}+v=2$$

$$\Rightarrow v=2-\frac{5}{3}$$

$$\Rightarrow v=\frac{6-5}{3}$$

$$\Rightarrow v=\frac{1}{3}$$

Now $\frac{1}{x-1}=\frac{1}{3}$ and $\frac{1}{y-2}=\frac{1}{3}$

=> $x - 1 = 3$ and $y - 2 = 3$

=> $x = 4$ and $y = 5$

Thus, the solution of the given pair of equations is $x = 4$ and $y = 5$.

Question 18 (3.0 marks)

The sum of 4th and 8th terms of an A.P. is 24 and sum of 6th and 10th terms is 44. Find A.P.

Solution:

Let a be the first term and d be the common difference of the A.P.

It is known that the n^{th} term of an A.P. is given by $a_n = a + (n-1)d$.

Fourth term, $a_4 = a + (4-1)d = a + 3d$
Eighth term, $a_8 = a + (8-1)d = a + 7d$
Sixth term, $a_6 = a + (6-1)d = a + 5d$
Tenth term, $a_{10} = a + (10-1)d = a + 9d$

Now, we have

$$a_4 + a_8 = 24$$
$$\Rightarrow a + 3d + a + 7d = 24$$
$$\Rightarrow 2a + 10d = 24$$
$$\Rightarrow a + 5d = 12 \qquad ...(1)$$

and

$$a_6 + a_{10} = 44$$
$$\Rightarrow a + 5d + a + 9d = 44$$
$$\Rightarrow 2a + 14d = 44$$
$$\Rightarrow a + 7d = 22 \qquad ...(2)$$

On subtracting equation (1) from equation (2), we obtain

$$(a + 7d) - (a + 5d) = 22 - 12$$
$$\Rightarrow 2d = 10$$
$$\Rightarrow d = 5$$

On substituting the value of d in equation (1), we obtain

$a + 5(5) = 12$
$\Rightarrow a = -13$

The A.P. is a, $a + d$, $a + 2d$ …

=> −13, −13 + 5, −13 + 10…

=> −13, −8, −3…

Thus, the A.P. is −13, −8, −3…

Question 19 (3.0 marks)

Construct a ΔABC in which BC = 6.5 cm, AB = 4.5 cm and ∠ABC = 60°. Construct a triangle similar to this triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

Solution:

A triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC can be drawn by following the given steps.

Step 1

Draw a 6.5 cm long line segment BC.

Step 2

Draw an arc of any radius while taking B as the centre. Let it intersect line BC at point O. Taking O as the centre, draw another arc to cut the previous arc at point O'. Join BO', which is the ray making 60° with line BC.

Step 3

Taking B as the centre, draw an arc of radius 4.5 cm intersecting the extended line segment BO' at point A. Join AC. In ΔABC, AB = 4.5 cm, BC = 6.5 cm, and ∠ABC = 60°.

Step 4

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 5

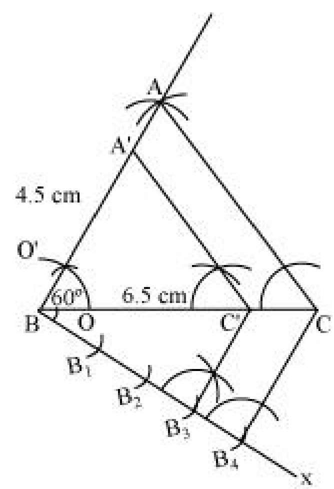
Locate 4 points (as 4 is greater in 3 and 4), B_1, B_2, B_3, and B_4 on line segment BX.

Step 6

Join B_4C and draw a line through B_3 such that it is parallel to B_4C and intersecting BC at C'.

Step 7

Draw a line through C' parallel to AC and intersecting AB at A'. ΔA'BC' is the required triangle.



Justification

The construction can be justified by proving

$$A'B = \frac{3}{4}AB,\ BC' = \frac{3}{4}BC,\ A'C' = \frac{3}{4}AC$$

In ΔA′BC′ and ΔABC,

□A′C′B = □ACB (Corresponding angles)

□A′BC′ = □ABC (Common)

=> ΔA′BC′ □ ΔABC (AA similarity criterion)

$$\Rightarrow \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} \quad \ldots (1)$$

In $\Delta BB_3C'$ and ΔBB_4C,

$\angle B_3BC' = \angle B_4BC$ (Common)

$\angle BB_3C' = \angle BB_4C$ (Alternate interior angles)

=> $\Delta BB_3C' \sim \Delta BB_4C$ (AA similarity criterion)

$$\Rightarrow \frac{BC'}{BC} = \frac{BB_3}{BB_4}$$

$$\Rightarrow \frac{BC'}{BC} = \frac{3}{4} \qquad ...(2)$$

From equations (1) and (2), we obtain

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{3}{4}$$

=> $A'B = \frac{3}{4}AB,\ BC' = \frac{3}{4}BC,\ A'C' = \frac{3}{4}AC$

This justifies the construction.

Question 20 (3.0 marks)

In Fig. 4, ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.

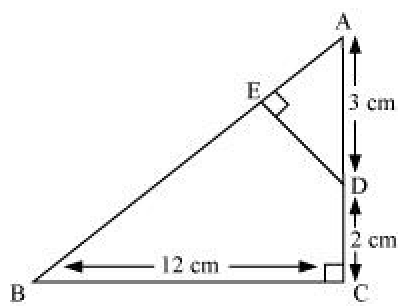


Fig. 4

OR

In Fig, 5, DEFG is a square and $\angle BAC = 90°$. Show that $DE^2 = BD \times EC$.

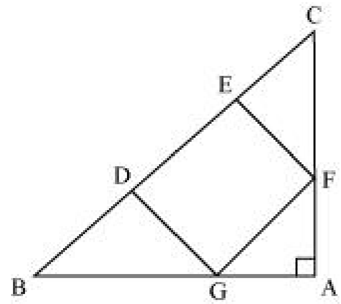
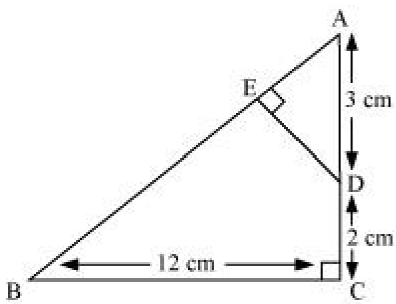


Fig. 5

Solution:



In ΔABC and ΔADE:

$\angle$ACB = $\angle$AED (= 90°)

$\angle$BAC = $\angle$DAE (Common)

=> ΔABC $\sim$ ΔADE (By AA similarity criterion)

It is known that corresponding sides of similar triangles are proportional.

$$\therefore \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$
$$\Rightarrow \frac{AB}{AD} = \frac{BC}{DE} = \frac{AD + DC}{AE}$$
$$\Rightarrow \frac{AB}{3\text{ cm}} = \frac{12\text{ cm}}{DE} = \frac{3\text{ cm} + 2\text{ cm}}{AE}$$
$$\Rightarrow \frac{AB}{3\text{ cm}} = \frac{12\text{ cm}}{DE} = \frac{5\text{ cm}}{AE} \qquad ...(1)$$

On applying Pythagoras theorem to ΔABC, we obtain

$AB^2 = AC^2 + BC^2$

=> $AB^2 = 5^2 + 12^2 = 25 + 144 = 169$

=> AB = 13 cm

On substituting the value of AB in equation (1), we obtain

$$\frac{13 \text{ cm}}{3 \text{ cm}} = \frac{12 \text{ cm}}{DE} = \frac{5 \text{ cm}}{AE} \quad \ldots(2)$$

$$\frac{13 \text{ cm}}{3 \text{ cm}} = \frac{12 \text{ cm}}{DE} \quad [\text{From } (2)]$$

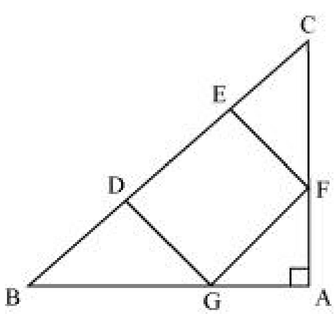
$$\Rightarrow DE = \frac{3 \times 12}{13} = \frac{36}{13} \approx 2.77 \text{ cm}$$

$$\text{Also, } \frac{13 \text{ cm}}{3 \text{ cm}} = \frac{5 \text{ cm}}{AE} \quad [\text{From } (2)]$$

$$\Rightarrow AE = \frac{3 \times 5}{13} = \frac{15}{13} \approx 1.15 \text{ cm}$$

Thus, the lengths of AE and DE are approximately 1.15 cm and 2.77 cm respectively.

OR



DEFG is a square.

=> $\angle EDG = \angle FED = 90°$

Now, $\angle BDG + \angle EDG = 180°$ [Linear pair]

=> $\angle BDG + 90° = 180°$

=> $\angle BDG = 90°$

Similarly, $\angle CEF = 90°$

Let □DBG = α

By angle sum property in ΔBDG:

□DGB + □DBG + □BDG = 180°

=> □DGB = 90° − α

By angle sum property in ΔABC:

□ECF + □ABC + □BAC = 180° [□ECF = □BCA]

=> □ECF = 90° − α

By angle sum property in ΔFEC:

□CEF + □ECF + □CFE = 180°

=> □CFE = α

Now, in ΔBDG and ΔFEC:

□BDG = □FEC (= 90°)

□DBG = □EFC (= α)

By AA similarity criterion, ΔBDG □ ΔFEC

We know that corresponding sides of similar triangles are proportional.

$\therefore \frac{BD}{FE} = \frac{DG}{EC}$

$\Rightarrow \frac{BD}{DE} = \frac{DE}{EC}$ [In square DEFG, DE = DG = EF = GF]

=> $DE^2 = BD \times EC$

Thus, the given result is proved.

Question 21 (3.0 marks)

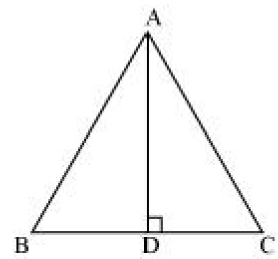
Find the value of sin 30° geometrically.

OR

Without using trigonometrical tables, evaluate:

$$\frac{\cos 58^\circ}{\sin 32^\circ}+\frac{\sin 22^\circ}{\cos 68^\circ}-\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \cdot \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$$

Solution:



Consider an equilateral ΔABC.

The measure of each interior angle of an equilateral triangle is 60°.

=>□BAC = □ACB = □CBA = 60°

Now, draw a perpendicular AD□ BC.

In an equilateral triangle, the perpendicular drawn from any vertex to the opposite side bisects the angle at that vertex and also bisects the other side.

=>□ BAD = □ CAD $=\frac{60^\circ}{2}$ = 30°

=> BD = DC

Now, ΔABD is a right triangle, right-angled at D with □BAD = 30º and

□ABD = 60º

Let the length of side AB be 2*a*.

Accordingly, $BD=\frac{BC}{2}=a$

Now, we have sin30º = $\frac{BD}{AB}=\frac{a}{2a}=\frac{1}{2}$

Thus, the value of sin 30º is $\frac{1}{2}$.

OR

$$\frac{\cos 58°}{\sin 32°}+\frac{\sin 22°}{\cos 68°}-\frac{\cos 38° \operatorname{cosec} 52°}{\tan 18° \tan 35° \tan 60° \tan 72° \tan 55°}$$
$$=\frac{\cos(90°-32°)}{\sin 32°}+\frac{\sin(90°-68°)}{\cos 68°}-\frac{\cos 38° \operatorname{cosec}(90°-38°)}{\tan 18° \tan 35° \tan 60° \tan(90°-18°) \tan(90°-35°)}$$
$$=\frac{\sin 32°}{\sin 32°}+\frac{\cos 68°}{\cos 68°}-\frac{\cos 38° \sec 38°}{\tan 18° \tan 35° \tan 60° \cot 18° \cot 35°}$$
$$[\sin(90°-\theta)=\cos\theta;\ \cos(90°-\theta)=\sin\theta;\ \operatorname{cosec}(90°-\theta)=\sec\theta;\ \tan(90°-\theta)=\cot\theta]$$
$$=1+1-\frac{1}{(\tan 18° \cot 18°)(\tan 35° \cot 35°)\tan 60°} \qquad [\cos\theta \sec\theta = 1]$$
$$=2-\frac{1}{1\times 1\times \tan 60°} \qquad [\tan\theta \cot\theta = 1]$$
$$=2-\frac{1}{\sqrt{3}} \qquad [\tan\theta = \frac{1}{\sqrt{3}}]$$
$$=\frac{2\sqrt{3}-1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{6-\sqrt{3}}{3}$$

Question 22 (3.0 marks)

Find the point on y-axis which is equidistant from the points (5, −2) and (−3, 2)

OR

The line segment joining the points A (2, 1) and B (5, −8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by
$2x - y + k = 0$, find the value of k.

Solution:

The required point is equidistant from points (5, −2) and (−3, 2).

Since the point lies on y-axis, its x-coordinate will be 0.

Thus, let its coordinates be (0, y).

The distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

=>Distance between (0, y) and (5, −2) = $\sqrt{(5-0)^2+(-2-y)^2}$

Similarly, distance between (0, y) and (−3, 2) = $\sqrt{(-3-0)^2+(2-y)^2}$

According to the given condition, $\sqrt{(5-0)^2+)(-2-y)^2} = \sqrt{(-3-0)^2+(2-y)^2}$

$$\Rightarrow (5-0)^2+(-2-y)^2=(-3-0)^2+(2-y)^2 \qquad \text{[Squaring both sides]}$$
$$\Rightarrow 25+4+y^2+4y=9+4+y^2-4y$$
$$\Rightarrow 8y=-16$$
$$\Rightarrow y=-2$$

Thus, the required point is (0, −2).

OR

P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of line segment AB, i.e.,

AP = PQ = QB

Therefore, point P divides AB internally in the ratio 1:2. Thus, by section formula,

$$x_1=\frac{1\times5+2\times2}{1+2},\ y_1=\frac{1\times(-8)+2\times1}{1+2}$$
$$x_1=3,\ y_1=-2$$
Therefore, the coordinates of point P are (3, − 2).

It is also given that point P (3, −2) lies on line $2x - y + k = 0$.

$$2(3)-(-2)+k=0$$
$$\Rightarrow 6+2+k=0$$
$$\Rightarrow k=-8$$

Thus, the value of k is −8.

Question 23 (3.0 marks)

If P (x, y) is any point on the line joining the points A (a, 0) and B (0, b), then show that
$\frac{x}{a}+\frac{y}{b}=1$.

Solution:

Since the three points A (a, 0), P (x, y), and C (0, b) lie on the same line, the area of the triangle formed by these three points is zero.

$$\frac{1}{2}[a(y-b)+x(b-0)+0(0-y)]=0$$
$$\Rightarrow a(y-b)+x(b-0)+0(0-y)=0$$
$$\Rightarrow ay-ab+xb=0$$
$$\Rightarrow ay+xb=ab$$
$$\Rightarrow \frac{ay}{ab}+\frac{xb}{ab}=1 \qquad \text{[Dividing both sides by } ab\text{]}$$
$$\Rightarrow \frac{y}{b}+\frac{x}{a}=1$$
$$\Rightarrow \frac{x}{a}+\frac{y}{b}=1$$

Hence, proved.

Question 24 (3.0 marks)

In Fig. 6, PQ = 24 cm, PR = 7 cm and O is the centre of the circle. Find the area of shaded region (take π = 3.14)

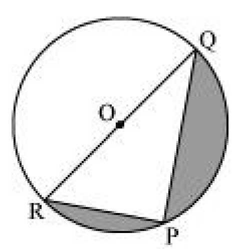


Fig. 6

Solution:

We know that the angle in a semicircle is a right angle.

=>□RPQ = 90°

Thus, ΔPQR represents a right-angled triangle where PR = 7 cm and PQ = 24 cm.

On applying Pythagoras theorem to right ΔPQR, we obtain

$QR^2 = PQ^2 + PR^2 = (24\text{ cm})^2 + (7\text{ cm})^2 = (576 + 49)\text{ cm}^2 = 625\text{ cm}^2$

=> QR = $\sqrt{625}$ cm = 25 cm

Area of semicircle = $\frac{1}{2}\pi r^2$, where $r = \frac{25}{2}\text{ cm} = 12.5\text{ cm}$

Area of semicircle = $\frac{1}{2} \times 3.14 \times 12.5 \times 12.5$ cm^2

$= \frac{1}{2} \times 3.14 \times 156.25\text{ cm}^2$

$= 245.3125\text{ cm}^2$

Area of ΔPQR = $\frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 7 \times 24\text{ cm}^2$ = 7 × 12 cm^2 = 84 cm^2

Thus, area of the shaded region

= Area of semicircle − Area of ΔPQR = 245.3125 − 84 = 161.3125 cm^2

Question 25 (3.0 marks)

The king, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (i) heart (ii) queen (iii) clubs.

Solution:

After removing the king, queen, and jack of clubs, the number of remaining cards is 49, out of which 13 are hearts, 13 are diamonds, 13 are spades, and 10 are clubs.

(i) Probability of drawing a heart $= \frac{\text{Total number of hearts}}{\text{Total number of remaining cards}} = \frac{13}{49}$

(ii) Probability of drawing a queen $= \frac{\text{Total number of remaining queens}}{\text{Total number of remaining cards}} = \frac{3}{49}$

(iii) Probability of drawing a club card $= \frac{\text{Total number of remaining club cards}}{\text{Total number of remaining cards}} = \frac{10}{49}$

Section D

Question Number 26 to 30 carry 6 marks each.

Question 26 (6.0 marks)

The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

OR

Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Solution:

Let the two odd numbers be $(2n - 1)$ and $(2n + 1)$, where n is a natural number.

It is given that $(2n - 1)^2 + (2n + 1)^2 = 394$

=> $4n^2 + 1 - 4n + 4n^2 + 1 + 4n = 394$

=> $8n^2 + 2 = 394$

=> $8n^2 = 392$

=> $n^2 = 49$

=> $n = 7$

=> $2n - 1 = 2\ (7) - 1 = 13$

=> $2n + 1 = 2\ (7) + 1 = 15$

Hence, the odd numbers are 13 and 15.

OR

Let the speed of car A be u and that of car B be v.

First condition (when both cars travel in the same direction)

Let both the cars meet at point C in 5 hours.

Car A travels distance AC, whereas car B travels distance BC.

Distance = time × speed

=> AC = 5 × u

BC = 5 × v

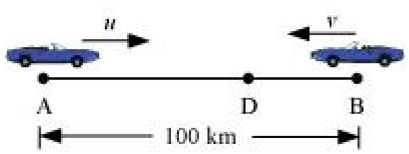
AC − BC = 100

=> $5u - 5v = 100$

=> $u - v = 20$ … (1)

Second condition (when both cars travel in opposite directions)

Let both cars meet at point D.



Car A will travel distance AD, whereas car B will travel distance BD.

Distance = time × speed

=> AD = 1 × u

=> BD = 1 × v

AD + BD = 100

=> $u + v = 100$ … (2)

On adding equations (1) and (2), we obtain

$2u = 120$

$\Rightarrow u = 60$

From equation (2), we obtain

$60 + v = 100$

$\Rightarrow v = 40$

Thus, the speed of car A is 60 km/h and that of car B is 40 km/h.

Question 27 (6.0 marks)

Prove that, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Using the above result, do the following:

In Fig. 7, DE||BC and BD = CE. Prove that ΔABC is an isosceles triangle.

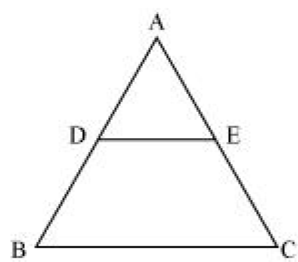
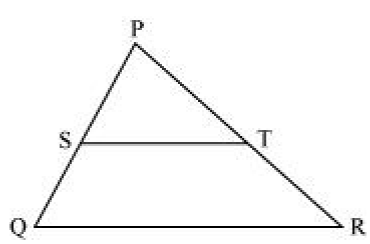


Fig. 7

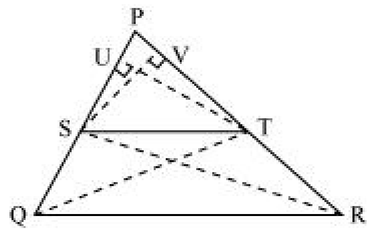
Solution:

Let us consider a ΔPQR. A line ST parallel to its base (i.e. QR) is drawn intersecting the sides PQ and PR at points S and T.



Now, line STdivides PQ into two parts i.e., PS and SQ. Line ST similarlydivides PR into two parts i.e., PT and TR.This information is sufficient to prove this theorem if we can prove that $\frac{PS}{SQ}=\frac{PT}{TR}$.

To do this, let usjoin QT and RS and draw TU PS and SV PT.



We know that area of triangle = $\frac{1}{2}\times \text{base}\times \text{height}$

=> $ar(\Delta PST)=\frac{1}{2}\times PS\times TU$

Taking PT as base and SV as height, we can write

$ar(\Delta PST)=\frac{1}{2}\times PT\times SV$

Similarly, $ar(\Delta QST)=\frac{1}{2}\times QS\times TU$

And, $ar(\Delta RST)=\frac{1}{2}\times TR\times SV$

Now, $\frac{ar(\Delta PST)}{ar(\Delta QST)}=\frac{\frac{1}{2}\times PS\times TU}{\frac{1}{2}\times QS\times TU}$

=> $\frac{ar(\Delta PST)}{ar(\Delta QST)}=\frac{PS}{QS}$ … (1)

Now, $\frac{ar(\Delta PST)}{ar(\Delta RST)} = \frac{\frac{1}{2} \times PT \times SV}{\frac{1}{2} \times TR \times SV}$

=> $\frac{ar(\Delta PST)}{ar(\Delta RST)} = \frac{PT}{TR}$ … (2)

ΔQST and ΔRST are on the same base i.e., ST and between the same parallel lines i.e., ST and QR.

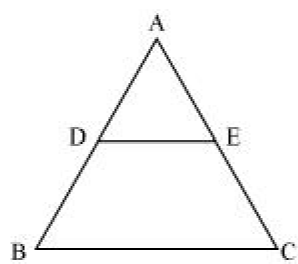
=>ar (ΔQST) = ar (ΔRST) … (3)

From equations (1), (2), and (3), we obtain

$\frac{PS}{QS} = \frac{PT}{TR}$

Hence, proved.

Now, consider ΔABC. Here, a line DE parallel to its base (i.e., BC) is drawn such that it intersects sides AB and BC at points D and E respectively.



By using the above result, we may find that

$\frac{AD}{BD} = \frac{AE}{CE}$

It is given that BD = CE … (4)

=>AD = AE … (5)

On adding equations (4) and (5), we obtain

BD + AD = CE + AE

=> AB = AC

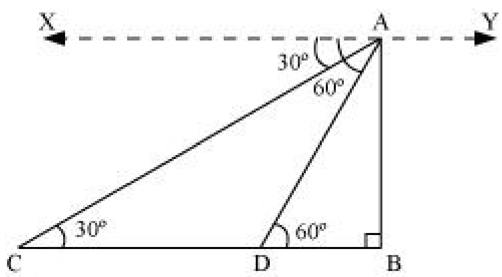
Thus, ΔABC is an isosceles triangle.

Question 28 (6.0 marks)

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Solution:

Let AB be a tower.



Initially, the car was at point C. After 6 s, it reaches point D.

Clearly, CD is the distance that the car travels in 6 s.

Here, ∠XAC is the angle of depression, when the car is at C, while ∠XAD is the angle of depression, when the car is at D.

∠ACB = ∠XAC = 30° (Alternate interior angles)

And, ∠ADB = ∠XAD = 60° (Alternate interior angles)

Now, in ΔABD,

$$\frac{AB}{BD} = \tan 60^\circ$$

$$\frac{AB}{BD} = \sqrt{3} \qquad ...(1)$$

In ΔABC,

$$\frac{AB}{BC} = \tan 30°$$

$$\frac{AB}{BC} = \frac{1}{\sqrt{3}} \qquad ...(2)$$

On dividing equation (1) by equation (2), we obtain

$$\frac{BC}{BD} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3$$

$$\frac{BD + DC}{BD} = 3$$

$$1 + \frac{DC}{BD} = 3$$

$$\frac{DC}{BD} = 2$$

$$BD = \frac{DC}{2} \qquad ...(3)$$

The car is approaching the foot of the tower with uniform speed (i.e., constant speed).

Speed = $\frac{\text{Distance}}{\text{Time}}$

Hence, $\frac{\text{Distance}}{\text{Time}}$ ratio will be constant each time.

Let the car take *t* seconds to reach the foot of the tower from this point.

$$\frac{DC}{6} = \frac{BD}{t}$$

$$\Rightarrow \frac{DC}{6} = \frac{DC}{2t} \qquad \text{[using equation (3)]}$$

=> $t = 3$ s

Thus, the car will reach the foot of the tower in 3 seconds.

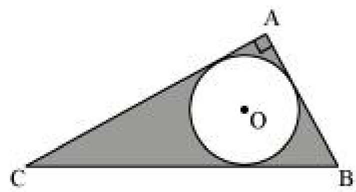
Question 29 (6.0 marks)

From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to two places of decimals. Also find the total surface area of the remaining solid. (take π = 3.1416)

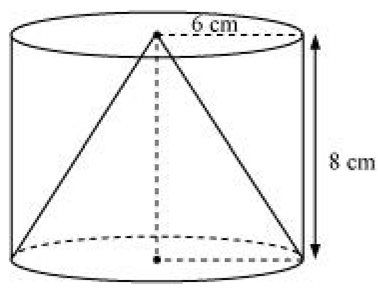
OR

In Fig. 8, ABC is a right triangle right angled at A. Find the area of shaded region if AB = 6 cm, BC = 10 cm and O is the centre of the incircle of ΔABC.

(take π = 3.14)



Solution:



Volume of the remaining solid = Volume of cylinder – Volume of conical cavity

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$
$$= \frac{2}{3}\pi r^2 h$$
$$= \frac{2}{3} \times 3.1416 \times (6\text{ cm})^2 \times (8\text{ cm})$$
$$= 603.1872\text{ cm}^3$$

Thus, the volume of the remaining solid up to two decimal places is

603.18 cm^3.

Now, the total surface area of the remaining solid is

Curved surface area of cylinder + area of top face + curved surface area of cone

$= 2\pi rh + \pi r^2 + \pi rl$

$= \pi r\,(2h + r + l)$

The slant height (l) of the cone is given by $\sqrt{r^2 + h^2}$.

=> Slant height of conical cavity = $\sqrt{(6\text{ cm})^2 + (8\text{ cm})^2} = 10\text{ cm}$

Therefore, total surface area of the solid

$= 3.1416 \times (6\text{ cm})\,(2 \times 8\text{ cm} + 6\text{ cm} + 10\text{ cm})$

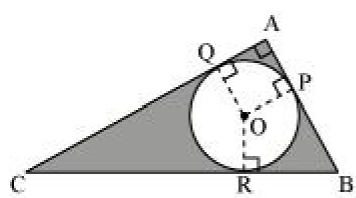
$= 3.1416 \times (6\text{ cm})\,(16\text{ cm} + 6\text{ cm} + 10\text{ cm})$

$= 3.1416 \times 6\text{ cm} \times 32\text{ cm}$

$= 603.1872\text{ cm}^2$

Thus, the total surface area of the remaining solid up to two decimal places is 603.18 cm^2.

OR



By Pythagoras theorem, in ΔABC, we obtain

$BC^2 = AC^2 + AB^2$

=> $AC^2 = BC^2 - AB^2 = (10\text{ cm})^2 - (6\text{ cm})^2$

=> $AC^2 = 64\text{ cm}^2$

=> AC = 8 cm

Let the radius of the incircle be r.

Let the circle touch side AB at P, side AC at Q, and side BC at R.

We know that the radius from the centre of the tangent is perpendicular to the tangent through the point of contact.

=> $OP \perp AB$, $OQ \perp AC$, and $OR \perp BC$

Also, the tangents drawn from an external point to the circle are equal.

=> AP = AQ (Tangents from point A)

BP = BR (Tangents from point B)

CR = CQ (Tangents from point C)

Now, in quadrilateral OPAQ, AQ = AP and □AQO = □APO = □PAQ = 90°

Therefore, OPAQ is a square.

=> OP = AQ = AP = OQ = r

=> AB = 6 cm − r

=> BR = 6 cm − r

CQ = 8 cm − r

=> CR = 8 − r

Now, BC = BR + CR

=> 10 cm = 6 cm − r + 8 cm − r

=> 10 cm = 14 cm − 2r

=> r = 2 cm

Thus, area of shaded region = Area of ΔABC − Area of circle

$= \frac{1}{2} \times AB \times AC - \pi r^2$

$= \frac{1}{2} \times (8\text{ cm}) \times (6\text{ cm}) - \pi\,(2\text{ cm})^2$

$= 24\text{ cm}^2 - 3.14 \times 4\text{ cm}^2$

$= 24\text{ cm}^2 - 12.56\text{ cm}^2$

$= 11.44\text{ cm}^2$

Thus, the area of the shaded region is 11.44 cm^2.

Question 30 (6.0 marks)

The following table gives the daily income of 50 workers of a factory:

Daily income (in Rs.)	100–120	120–140	140–160	160–180	180–200
Number of workers	12	14	8	6	10

Find the Mean, Mode and Median of the above data.

Solution:

Let the assumed mean a be 150.

The table for the given data can be drawn as

Class interval	**Number of workers (f_i)**	**Class mark (x_i)**	**$d_i = x_i - 150$**	**$f_i d_i$**	**Cumulative frequency (cf)**
100–120	12	110	–40	–480	12
120–140	14	130	–20	–280	26
140–160	8	150	0	0	34
160–180	6	170	20	120	40
180–200	10	190	40	400	50
Total	50			–240	

Mean is given by $a + \dfrac{\sum f_i d_i}{\sum f_i}$

$$\therefore \text{Mean} = 150 + \frac{(-240)}{50}$$
$$= 150 + (-4.8) = 145.2$$

Thus, the mean of the given data is 145.2.

It can be seen in the data table that the maximum frequency is 14. The class corresponding to this frequency is 120–140.

=> Modal class = 120 – 140

Lower limit of modal class (l) = 120

Class size (h) = 140 − 120 = 20

Frequency of modal class (f_1) = 14

Frequency of class preceding the modal class (f_1) = 12

Frequency of class succeeding the modal class (f_1) = 8

Mode is given by $l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

$$\begin{aligned}\therefore \text{Mode} &= 120+\left(\frac{14-12}{2\times 14-12-8}\right)\times 20\\ &= 120+\left(\frac{2}{28-12-8}\right)\times 20\\ &= 120+\left(\frac{2}{8}\right)\times 20\\ &= 120+5\\ &= 125\end{aligned}$$

Thus, the mode of the given data is 125.

Here, number of observations (n) = 50

$$\therefore \frac{n}{2}=\frac{50}{2}=25$$

This observation lies in class interval 120–140.

Therefore, the median class is 120–140.

Lower limit of median class (l) = 120

Cumulative frequency of class preceding the median class (cf) = 12

Frequency of median class (f) = 14

Median of the data is given by $l+\left(\dfrac{\frac{n}{2}-\text{cf}}{f}\right)\times h$

$$\therefore \text{Median}=120+\left(\frac{25-12}{14}\right)\times 20$$
$$=120+\frac{13}{14}\times 20$$
$$=120+\frac{130}{7}$$
$$=120+18.57$$
$$=138.57$$

Thus, the median of the given data is 138.57.