Section A

1. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives a rational or an irrational number.

Solution:

The given expression $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ can be simplified as:

$$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{3\times3\times5}+3\sqrt{2\times2\times5}}{2\sqrt{5}}$$

$$= \frac{2\times3\times\sqrt{5}+3\times2\times\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{12\sqrt{5}}{2\sqrt{5}}$$

$$= 6$$

Since 6 is a rational number, the expression $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives a rational number.

2. If α, β are the zeroes of the polynomial $2y^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$.

Solution:

Since α and β are the zeroes of the polynomial $2y^2 + 7y + 5$:

$$\alpha+\beta = \frac{-(\text{coefficient of } y)}{\text{coefficient of } y^2} = \frac{-7}{2}$$

$$\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } y^2} = \frac{5}{2}$$

$$\therefore\ \alpha + \beta + \alpha\beta = \frac{-7}{2} + \frac{5}{2} = \frac{-2}{2} = -1$$

Thus, the value of $\alpha + \beta + \alpha\beta$ is –1.

3. If the sum of the first q terms of an A.P. is $2q + 3q^2$, what is its common difference?

Solution:

Let a and d respectively be the first term and common difference of the A.P.

It is given that $S_q = 2q + 3q^2$

$\therefore\ S_1 = 2 \times 1 + 3 \times 1^2 = 5$

$\Rightarrow a = S_1 = 5$

$S_2 = 2 \times 2 + 3 \times 2^2 = 4 + 12 = 16$

$\Rightarrow a_1 + a_2 = 16$

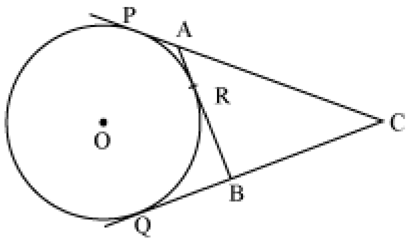
$\Rightarrow a + (a + d) = 16$

$\Rightarrow 2 \times 5 + d = 16$

$\Rightarrow d = 16 - 10 = 6$

Thus, the common difference of the A.P. is 6.

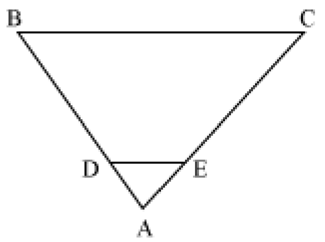
4. In Figure 1, CP and CQ are tangents from an external point C to a circle with centre O. AB are another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC.



Solution:
It is known that tangents drawn from an external point to a circle are of equal length.
BR = BQ = 4 cm ... (i)
CQ = CP = 11 cm
$\Rightarrow$ CQ = 11 cm
$\Rightarrow$ CB + BQ = 11 cm
$\Rightarrow$ CB + BR = 11 cm {using equation (i)}
$\Rightarrow$ BC + 4 cm = 11 cm
$\Rightarrow$ BC = 11 cm – 4 cm = 7 cm

Thus, the length of BC is 7 cm.

5. In Figure 2, DE||BC in ΔABC such that BC = 8 cm, AB = 6 cm and DA = 1.5 cm. Find DE.



Solution:
In ΔABC and ΔADE:
$\angle ABC = \angle ADE$ {Corresponding angles}
$\angle BAC = \angle DAE$ {Common}
$\therefore\ \Delta ABC \sim \Delta ADE$ {By AA Similarity Criterion}

It is known that corresponding sides of similar triangle are proportional.

$$\therefore \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{AB}{AD} = \frac{BC}{DE}$$

$$\Rightarrow \frac{6\text{ cm}}{1.5\text{ cm}} = \frac{8\text{ cm}}{DE}$$

$$\Rightarrow DE = \frac{8 \times 1.5}{6}\text{ cm}$$

$\therefore$ DE = 2 cm

Thus, the length of DE is 2 cm.

6. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, find the value of $5\left(x^2 - \frac{1}{x^2}\right)$.

Solution:

$$5x = \sec\theta$$

$$\Rightarrow x = \frac{\sec\theta}{5}$$

$$\frac{5}{x} = \tan\theta$$

$$\Rightarrow \frac{1}{x} = \frac{\tan\theta}{5}$$

$$\therefore 5\left(x^2 - \frac{1}{x^2}\right) = 5\left[\left(\frac{\sec\theta}{5}\right)^2 - \left(\frac{\tan\theta}{25}\right)^2\right]$$

$$= 5\left[\frac{\sec^2\theta}{25} - \frac{\tan^2\theta}{25}\right]$$

$$= \frac{5}{25}\left[\frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}\right]$$

$$= \frac{1}{5}\left[\frac{1-\sin^2\theta}{\cos^2\theta}\right]$$

$$= \frac{1}{5}\left(\frac{\cos^2\theta}{\cos^2\theta}\right) \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{5}$$

Thus, the value of the given expression is $\frac{1}{5}$

7. What is the distance between the points A (c, 0) and B(0, –c)?

Solution:

Using distance formula, the distance between the points A (c, 0), and B (0, –c) is given by:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0-c)^2 + (-c-0)^2} \text{ units}$$

$$= \sqrt{c^2 + c^2} \text{ units}$$

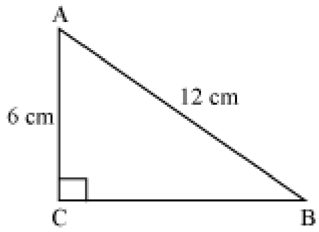
$$= \sqrt{2c^2} \text{ units}$$

$$= c\sqrt{2} \text{ units}$$

Thus, the distance between the given points is $c\sqrt{2}$ units.

8. In a ΔABC, right-angled at C, AC = 6 cm and AB = 12 cm. Find ∠A.

Solution:



In ΔABC, $\cos A = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

$\therefore \cos A = \frac{AC}{AB} = \frac{6\text{ cm}}{12\text{ cm}} = \frac{1}{2} = \cos 60°$

Thus, the measure of ∠A is 60°.

9. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, write the height of the frustum.

Solution:
Let h be the height of the frustum of the cone.
Let r_1 and r_2 be the radii of the ends of the frustum.
$\therefore r_1 - r_2 = 4$ cm
Slant height, $l = 5$ cm
The slant height of the frustum of a cone is given by, $l^2 = h^2 + (r_1 - r_2)^2$
$\therefore h^2 = l^2 - (r_1 - r_2)^2$
$\Rightarrow h^2 = (5\text{ cm})^2 - (4\text{ cm})^2$
$\Rightarrow h^2 = 25\text{ cm}^2 - 16\text{ cm}^2 = 9\text{ cm}^2$
$\therefore h = 3$ cm

Thus, the height of the frustum of the cone is 3 cm.

10. A die is thrown once. What is the probability of getting a number greater than 4?

Solution:
When a die is thrown, there are 6 possible outcomes {1, 2, 3, 4, 5, 6}
The numbers greater than 4 are 5 and 6.
∴ Number of favourable outcomes = 2
$\therefore$ Required probability $= \frac{2}{6} = \frac{1}{3}$

Thus, the probability of getting a number greater than 4 is $\frac{1}{3}$.

Section B

11. For what value of k, is 3 a zero of the polynomial $2x^2 + x + k$?

Solution:
It is given that 3 is a zero of the polynomial $p(x) = 2x^2 + x + k$
$\therefore\ p(3) = 0$
$\Rightarrow 2(3)^2 + 3 + k = 0$
$\Rightarrow 18 + 3 + k = 0$
$\Rightarrow k = -21$
Thus, the value of k is –21.

12. Find the value of m for which the pair of linear equations $2x + 3y - 7 = 0$ and $(m - 1)\,x + (m + 1)\,y = (3m - 1)$ has infinitely many solutions.

Solution:
The given pair of linear equations is:
$2x + 3y - 7 = 0$
$(m - 1)\,x + (m + 1)\,y = (3m - 1)$
$\Rightarrow (m - 1)\,x + (m + 1)\,y - (3m - 1) = 0$

It is known that the pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinitely many solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$
$a_2 = m-1,\ b_2 = m+1,\ c_2 = -(3m-1)$

$$\therefore \frac{2}{m-1} = \frac{3}{m+1} = \frac{-7}{-(3m-1)}$$

$$\frac{2}{m-1} = \frac{3}{m+1}$$

$\Rightarrow 2(m + 1) = 3(m - 1)$
$\Rightarrow 2m + 2 = 3m - 3$
$\Rightarrow 3m - 2m = 2 + 3$
$\Rightarrow m = 5$

Thus, the value of m is 5.

13. Find the common difference of an A.P. whose first term is 4, the last term is 49 and the sum of all its terms is 265.

Solution:
It is known that the sum of n terms of an AP whose first term is a and last term is l is given as $S_n = \frac{n(a+l)}{2}$
Here, first term, $a = 4$
Last term, $l = 49$
Sum of n terms $(S_n) = 265$

$\therefore 265 = \frac{n(4+49)}{2}$

$\Rightarrow 265 = \frac{n \times 53}{2}$

$\Rightarrow n = \frac{265 \times 2}{53} = 10$

Therefore, the series has 10 terms.

It is known that the n^{th} term of an AP is given by, $a_n = a + (n - 1)\,d$

$\therefore\ 49 = 4 + (10 - 1)\,d$

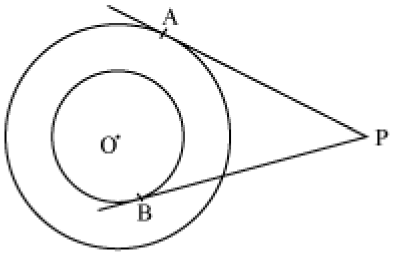
$\Rightarrow 49 - 4 = 9d$

$\Rightarrow 45 = 9d$

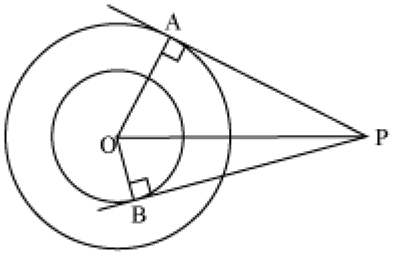
$\Rightarrow d = 5$

Thus, the common difference of the AP is 5.

14. In figure 3, there are two concentric circles with centre O and of radii 5 cm and 3 cm. From an external point P, Tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.



Solution:
Firstly, OA, OB and OP are joined.



$\therefore$ OA = 5 cm
OB = 3 cm

It is known that the radius is perpendicular to the tangent at the point of contact.

$\therefore$ OA $\perp$ PA and OB $\perp$ PB

Therefore, ΔOAP and ΔOBP are right triangles, right-angled at A and B respectively.

Applying Pythagoras Theorem in ΔOAP:

$OP^2 = AP^2 + OA^2$

$\Rightarrow OP^2 = (12\text{ cm})^2 + (5\text{ cm})^2$

$\Rightarrow OP^2 = 144\text{ cm}^2 + 25\text{ cm}^2$

$\Rightarrow OP^2 = 169\text{ cm}^2$

$\therefore OP = \sqrt{169}\text{ cm} = 13\text{ cm}$

Now, applying Pythagoras Theorem in ΔOBP:
$OP^2 = BP^2 + OB^2$
$\Rightarrow BP^2 = OP^2 - OB^2$
$\Rightarrow BP^2 = (13\text{ cm})^2 - (3\text{ cm})^2$
$\Rightarrow BP^2 = 169\text{ cm}^2 - 9\text{ cm}^2$
$\Rightarrow BP^2 = 160\text{ cm}^2$
$\therefore BP = \sqrt{160}\text{ cm} = 4\sqrt{10}\text{ cm}$

Thus, the length of BP is $4\sqrt{10}$ cm

15. Without using trigonometric tables, evaluate the following:

$$\frac{\cos 70^\circ}{3\sin 20^\circ} + \frac{4(\sec^2 59^\circ - \cot^2 31^\circ)}{3} - \frac{2}{3}\sin 90^\circ$$

Solution:

$$\frac{\cos 70^\circ}{3\sin 20^\circ} + \frac{4(\sec^2 59^\circ - \cot^2 31^\circ)}{3} - \frac{2}{3}\sin 90^\circ$$

$$= \frac{\cos(90^\circ - 20^\circ)}{3\sin 20^\circ} + \frac{4\{\sec^2(90^\circ - 31^\circ) - \cot^2 31^\circ\}}{3} - \frac{2}{3}\times 1 \qquad \{\because \sin 90^\circ = 1\}$$

$$= \frac{\sin 20^\circ}{3\sin 20^\circ} + \frac{4(\operatorname{cosec}^2 31^\circ - \cot^2 31^\circ)}{3} - \frac{2}{3}$$

$$\{\because \cos(90^\circ - \theta) = \sin\theta, \sec(90^\circ - \theta) = \operatorname{cosec}\theta\}$$

$$= \frac{1}{3} + \frac{4\times 1}{3} - \frac{2}{3} \qquad \{\because \operatorname{cosec}^2\theta - \cot^2\theta = 1\}$$

$$= \frac{1+4-2}{3}$$

$$= \frac{3}{3}$$

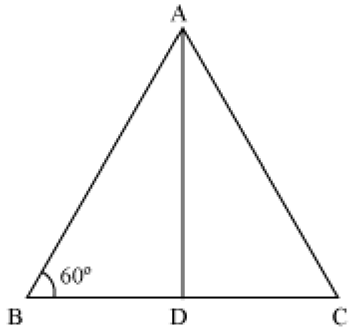
$$= 1$$

Thus, the value of the given expression is 1.

OR

Find the value of sec 60° geometrically.

Solution:



Consider an equilateral triangle ABC.
Each angle in an equilateral triangle is 60°.
Therefore, $\angle A = \angle B = \angle C = 60°$
Let the perpendicular AD be drawn from A to side BC.

Let the length of AB be $2a$.

$$\therefore \text{BD} = \frac{1}{2}\text{BC} = a$$

$$\text{Now, } \cos 60° = \frac{\text{BD}}{\text{AB}} = \frac{a}{2a} = \frac{1}{2}$$

$$\therefore \sec 60° = \frac{1}{\cos 60°} = \frac{1}{\frac{1}{2}} = 2$$

Thus, the value of sec 60° is 2.

Section C

16. Prove that $\sqrt{3}$ is an irrational number.

Solution:

If possible, suppose that $\sqrt{3}$ is a rational number.

Then, there exists integers p, q ($q \neq 0$, and p, q are co-prime) such that $\sqrt{3} = \frac{p}{q}$

On squaring both the sides, $3 = \frac{p^2}{q^2}$

$\Rightarrow p^2 = 3q^2$ ... (i)

Therefore, p^2 is a multiple of 3.

So, p is a multiple of 3.

Let $p = 3r$ where r is an integer

On squaring both sides:

$p^2 = 9r^2$

$\Rightarrow 3q^2 = 9r^2$ [using equation (i)]

$\Rightarrow q^2 = 3r^2$

Therefore, q^2 is a multiple of 3

So, q is multiple of 3

Now, p and q are multiples of 3.

So, the numbers p and q are not co-prime, which is a contradiction.

Thus, our assumption that $\sqrt{3}$ is a rational number is wrong.

Hence, $\sqrt{3}$ is an irrational number.

17. Solve the following pair of linear equations for x and y:

$$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$$
$$x + y = 2ab$$

Solution:

The given pair of linear equations is:

$$\frac{b}{a}x + \frac{a}{b}y = (a^2 + b^2) \quad ...(1)$$
$$x + y = 2ab \quad ...(2)$$

The given equations can be rewritten as:

$$\frac{b}{a}x + \frac{a}{b}y - (a^2 + b^2) = 0 \quad ...(3)$$
$$x + y - 2ab = 0 \quad ...(4)$$

Using cross multiplication method, equations (3) and (4) can be solved as:

$$\frac{x}{\left\{\frac{a}{b}\times(-2ab)\right\}-\{-(a^2+b^2)\times 1\}}=\frac{y}{\{-(a^2+b^2)\times 1\}-\left\{\frac{b}{a}\times(-2ab)\right\}}=\frac{1}{\left(\frac{b}{a}\times 1\right)-\left(\frac{a}{b}\times 1\right)}$$

$$\Rightarrow \frac{x}{-2a^2+a^2+b^2}=\frac{y}{-a^2-b^2+2b^2}=\frac{1}{\frac{b}{a}-\frac{a}{b}}$$

$$\Rightarrow \frac{x}{-a^2+b^2}=\frac{y}{-a^2+b^2}=\frac{ab}{b^2-a^2}$$

$$\therefore x=\frac{(b^2-a^2)(ab)}{b^2-a^2}=ab$$

$$y=\frac{(b^2-a^2)(ab)}{b^2-a^2}=ab$$

Thus, the solution of the given pair of linear equations is $x = ab$ and $y = ab$.

OR

The sum of the numerator and the denominator of a fraction is 4 more than twice the numerator. If 3 is added to each of the numerator and denominator, their ratio becomes 2:3. Find the fraction.

Solution:

Let the fraction be $\frac{x}{y}$.

It is given that the sum of the numerator and the denominator of the fraction is 4 more than twice the numerator.

$\therefore x + y = 2x + 4$

$\Rightarrow x = y - 4$ … (1)

It is given that if 3 is added to each of the numerator and denominator, their ratio becomes 2 : 3.

$$\therefore \frac{x+3}{y+3}=\frac{2}{3}$$

$$\Rightarrow \frac{y-4+3}{y+3}=\frac{2}{3} \qquad \{\text{using equation (1)}\}$$

$$\Rightarrow \frac{y-1}{y+3}=\frac{2}{3}$$

$$\Rightarrow 3y-3=2y+6$$

$$\Rightarrow 3y-2y=6+3$$

$$\Rightarrow y=9$$

$$\therefore x=y-4=9-4=5$$

Thus, the required fraction is $\frac{5}{9}$.

18. In an A.P., the sum of its first ten terms is – 80 and the sum of its next ten terms is – 280. Find the A.P.

Solution:

Let a and d respectively be the first term and the common difference of the A. P.

It is known that the sum of n terms of an A.P is given by $S_n = \frac{n}{2}[2a+(n-1)d]$

It is given that the sum of the first 10 terms, S_{10} is –80 and the sum of the next 10 terms is –280.

$\therefore$ S_{20} = –80 + (–280) = –360

$S_{10} = \frac{10}{2}[2a+(10-1)d] = -80$

$\Rightarrow 5(2a+9d) = -80$

$\Rightarrow 2a+9d = -16$ ... (1)

$S_{20} = \frac{20}{2}[2a+(20-1)d] = -360$

$\Rightarrow 10(2a+19d) = -360$

$\Rightarrow 2a+19d = -36$ ... (2)

Subtracting equation (1) from (2):

$10d$ = –36 + 16 = –20

$\Rightarrow d = -2$

Substituting d = –2 in equation (1):

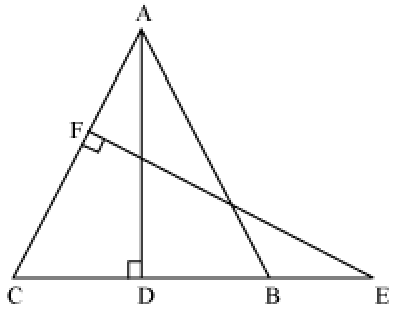
$2a$ = –16 – 9 (–2) = –16 + 18 = 2

$\therefore$ a = 1

Thus, the required A.P. is 1, –1, –3, – 5 …

19. In figure 4, ABC is an isosceles triangle in which AB = AC. E is a point on the side CB produced, such that FE ⊥ AC. If AD ⊥ CB, prove that:

AB × EF = AD × EC



Solution:

It is given that the length of AB is same as that of AC.

It is known that angles opposite to equal sides of a triangle are equal.

$\therefore$ ∠ACB = ∠ABC

$\Rightarrow$ ∠FCE = ∠ABD ... (1)

In ΔADB and ΔEFC:

∠ADB = ∠EFC = 90°

∠ABD = ∠ECF {Using (1)}

$\therefore$ ΔADB ~ ΔEFC (By AA similarity criterion)

It is known that the corresponding sides of similar triangles are in proportion.

$\therefore \frac{AB}{EC} = \frac{AD}{EF}$
$\Rightarrow AB \times EF = AD \times EC$

Thus, the given result is proved.

20. Prove the following:

(1 + cot A – cosec A) (1 + tan A + sec A) = 2

Solution:

$$\text{L.H.S.} = (1+\cot A - \text{cosec}A)(1+\tan A+\sec A)$$
$$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right) \qquad \left[\cot A = \frac{\cos A}{\sin A},\ \tan A = \frac{\sin A}{\cos A}\right]$$
$$= \left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$$
$$= \frac{(\sin A+\cos A-1)(\sin A+\cos A+1)}{\sin A\cos A}$$
$$= \frac{(\sin A+\cos A)^2-1^2}{\sin A\cos A} \qquad [(x+y)(x-y)=x^2-y^2]$$
$$= \frac{\sin^2 A+\cos^2 A+2\sin A\cos A-1}{\sin A\cos A}$$
$$= \frac{1+2\sin A\cos A-1}{\sin A\cos A} \qquad [\sin^2 A+\cos^2 A=1]$$
$$= 2 = \text{R.H.S.}$$

$\therefore$ (1 + cot$A$ – cosec$A$) (1 + tan$A$ + sec A) = 2

OR

Prove the following:
sin A (1 + tan A) + cos A (1 + cot A) = sec A + cosec A

Solution:

$$\text{L.H.S.} = \sin A(1+\tan A)+\cos A(1+\cot A)$$

$$= \sin A\left(1+\frac{\sin A}{\cos A}\right)+\cos A\left(1+\frac{\cos A}{\sin A}\right) \qquad \left[\tan A=\frac{\sin}{\cos A},\ \cot A=\frac{\cos A}{\sin A}\right]$$

$$= \sin A\left(\frac{\cos A+\sin A}{\cos A}\right)+\cos A\left(\frac{\sin A+\cos A}{\sin A}\right)$$

$$= (\cos A+\sin A)\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)$$

$$= (\cos A+\sin A)\left(\frac{\sin^2 A+\cos^2 A}{\cos A\sin A}\right)$$

$$= (\cos A+\sin A)\left(\frac{1}{\cos A\sin A}\right) \qquad [\sin^2 A+\cos^2 A=1]$$

$$= \frac{\cos A+\sin A}{\cos A\sin A}$$

$$= \frac{1}{\sin A}+\frac{1}{\cos A}$$

$$= \operatorname{cosec} A+\sec A$$

$$= \sec A+\operatorname{cosec} A = \text{R.H.S.}$$

$\therefore$ $\sin A(1 + \tan A) + \cos A(1 + \cot A) = \sec A + \operatorname{cosec} A$

21. Construct a triangle ABC in which AB = 8 cm, BC = 10 cm and AC = 6 cm. Then construct another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of ΔABC.

Solution:
The steps of construction are as follows:

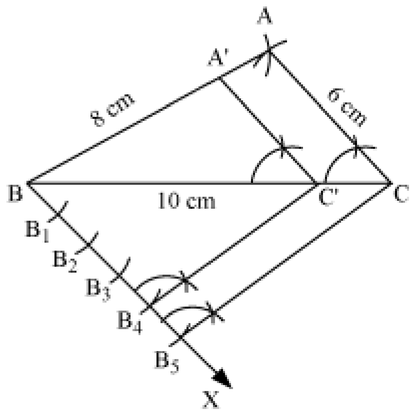
Step 1:
Draw a line segment BC = 10 cm. Taking point B as centre and radius equal to 8 cm, draw an arc. Similarly, taking point C as centre and radius equal to 6 cm, draw an arc. These arcs will intersect each other at point A. Join AB, AC. ΔABC is thus formed.

Step 2:
Draw a ray BX making an acute angle with line BC on the opposite side of vertex A.

Step 3:
Mark 5 points B_1, B_2, B_3, B_4, B_5 (as 5 is greater between 4 and 5) on ray BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$

Step 4:
Join CB_5. Through point B_4, draw a line parallel to CB_5 to intersect BC at point C'.

Step 5:
Through point C', draw a line parallel to line AC to intersect AB at A'.
Thus, ΔA'BC' is the required triangle.



22. Point P divides the line segment joining the points A (–1, 3) and B (9, 8) such that $\frac{\text{AP}}{\text{PB}} = \frac{k}{1}$. If P lies on the line $x - y + 2 = 0$, find the value of k.

Solution:

Since $\frac{\text{AP}}{\text{PB}} = k:1$, point P divides the line segment AB in the ratio k: 1.

Using section formula, the coordinates of point P is given by:

$$\left(\frac{9k+1(-1)}{k+1}, \frac{8k+1(3)}{k+1}\right)$$

$$= \left(\frac{9k-1}{k+1}, \frac{8k+3}{k+1}\right)$$

Since point P lies on the line $x - y + 2 = 0$:

$$\frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0$$

$\Rightarrow 9k - 1 - 8k - 3 + 2k + 2 = 0$

$\Rightarrow 3k - 2 = 0$

$\Rightarrow k = \frac{2}{3}$

Thus, the value of k is $\frac{2}{3}$.

23. If the points (p, q); (m, n) and $(p - m, q - n)$ are collinear, show that $pn = qm$.

Solution:

It is known that the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) will be collinear if:

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

It is given that the points (p, q), (m, n) and $(p - m, q - n)$ are collinear.

$\therefore\ p\{n-(q-n)\}+m(q-n-q)+(p-m)(q-n)=0$

$\Rightarrow\ p(n-q+n)+m(-n)+(pq-mq-np+mn)=0$

$\Rightarrow\ p(2n-q)+m(-n)+(pq-mq-np+mn)=0$

$\Rightarrow 2np-pq-mn+pq-mq-np+mn=0$

$\Rightarrow (2np-np)+(-pq+pq)-mq+(-mn+mn)=0$

$\Rightarrow np-mq=0$

$\Rightarrow np=mq$

Thus, the given result is proved.

24. The rain-water collected on the roof of a building, of dimensions 22 m × 20 m, is drained into a cylindrical vessel having base diameter 2 m and height 3.5 m. If the vessel is full up to the brim, find the height of rain-water on the roof [Use $\pi = \frac{22}{7}$]

Solution:
Let the height of the rain water on the roof be h
Length of the roof, l = 22 m
Breadth of roof, b = 20 m
Diameter of the cylindrical vessel = 2 m

$\therefore$ Radius (r) of the cylindrical vessel $\frac{2\text{ m}}{2}$ = 1 m

Height (H) of the cylindrical vessel = 3.5 m

Since the vessel is full of rain water up to the brim, it can be concluded that:
Volume of rain water on the roof = volume of the cylindrical vessel

$\Rightarrow l\times b\times h=\pi r^2 H$

$\Rightarrow 22\text{ m}\times 20\text{ m}\times h=\frac{22}{7}\times(1\text{ m})^2\times 3.5\text{ m}$

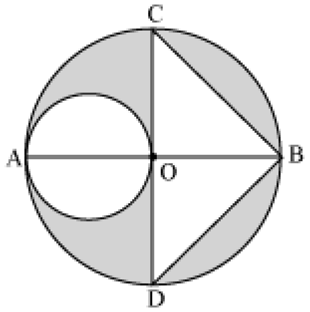
$\Rightarrow h=\frac{22\times 1\times 3.5}{7\times 22\times 20}\text{ m}$

$\Rightarrow h=\frac{1}{40}\text{ m}=\frac{1}{40}\times 100\text{ cm}=2.5\text{ cm}$

Thus, the height of rain water on the roof is 2.5 cm.

OR

In figure 5, AB and CD are two perpendicular diameters of a circle with centre O. If OA = 7 cm, find the area of the shaded region. [Use $\pi = \frac{22}{7}$]



Solution:
It can be seen in the given figure that, OA = OB = OC = OD = radius of the circle = 7 cm.
$\therefore$ CD = OC + OD = 7 cm + 7 cm = 14 cm
It is given that AB and CD are perpendicular diameters.
Therefore, OB is the altitude of ΔCDB corresponding to side CD.
Now, radius of the smaller circle $= \frac{OA}{2}$
Area of shaded region = Area of the bigger circle – Area of smaller circle – Area of ΔCDB

$$= \pi(OA)^2 - \pi\left(\frac{OA}{2}\right)^2 - \frac{1}{2}\times CD\times OB$$

$$= \frac{22}{7}\times (7\text{ cm})^2 - \frac{22}{7}\times\left(\frac{7\text{ cm}}{2}\right)^2 - \frac{1}{2}\times 14\text{ cm}\times 7\text{ cm}$$

$$= \left(22\times 7 - \frac{22}{7}\times\frac{49}{4} - 7\times 7\right)\text{ cm}^2$$

$$= \left(154 - \frac{77}{2} - 49\right)\text{ cm}^2$$

$$= \left(105 - \frac{77}{2}\right)\text{ cm}^2$$

$$= \left(\frac{210-77}{2}\right)\text{ cm}^2$$

$$= \frac{133}{2}\text{ cm}^2$$

$$= 66.5\text{ cm}^2$$

Thus, the area of the shaded region is 66.5 cm^2.

25. A bag contains cards which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that it bears
(i) a two digit number,
(ii) a number which is a perfect square.

Solution:
The bag contains cards which are numbered from 2 to 90.
Number of cards in the bag = 90 – (2 – 1) = 89

(i) The two-digit numbers from 2 to 90 are 10, 11, 12, 13 ... 90
Number of cards in the bag that bear a two-digit number = 81
$\therefore$ Required probability = $\frac{81}{89}$

Thus, the probability that a randomly drawn card bears a two-digit number is $\frac{81}{89}$.

(ii) The perfect squares from 2 to 90 are 4, 9, 16, 25, 36, 49, 64 and 81.
Number of cards in the bag that bear a number that is a perfect square = 8
$\therefore$ Required probability $= \frac{8}{89}$

Thus, the probability that a randomly drawn card bears a perfect square is $\frac{8}{89}$.

Section D

26. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

Solution:
Let the present age of the girl be x years.

$\therefore$ Present age of the girl's sister = $\frac{x}{2}$ years

After four years:
Age of girl = $(x + 4)$ years

Age of girl's sister = $\left(\frac{x}{2}+4\right)$ years

It is given that four years hence, the product of ages of the girl and her sister is 160.

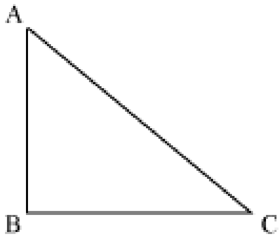
$$\therefore (x+4)\left(\frac{x}{2}+4\right)=160$$
$$\Rightarrow \frac{(x+4)(x+8)}{2}=160$$
$$\Rightarrow x^2+8x+4x+32=2\times160$$
$$\Rightarrow x^2+12x+32-320=0$$
$$\Rightarrow x^2+12x-288=0$$
$$\Rightarrow x^2+24x-12x-288=0$$
$$\Rightarrow x(x+24)-12(x+24)=0$$
$$\Rightarrow (x-12)(x+24)=0$$
$$\Rightarrow x-12=0 \text{ or } x+24=0$$
$$\Rightarrow x=12 \text{ or } -24$$

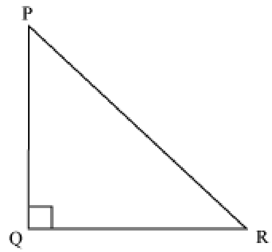
Since age of the girl cannot be negative, x = 12

Thus, the present age of the girl is 12 years and that of her sister is $\frac{12}{2}$ years $= 6$ years.

27. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.
Using the above, do the following:
In an isosceles triangle PQR, PQ = QR and $PR^2 = 2\ PQ^2$. Prove that $\angle$Q is a right angle.

Solution:
Suppose we are given ΔABC in which $AC^2 = AB^2 + BC^2$. We have to prove that $\angle B = 90°$
Let us construct ΔPQR right-angled at Q such that PQ = AB and QR = BC





Applying Pythagoras theorem in ΔPQR:
$PR^2 = PQ^2 + QR^2$
$\Rightarrow PR^2 = AB^2 + BC^2$ … (1) (By construction)

However, $AC^2 = AB^2 + BC^2$ … (2) (Given)
From (1) and (2), we obtain:
AC = PR … (3)

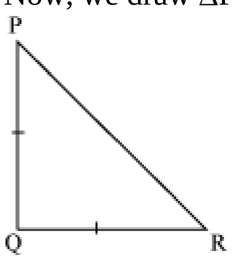
Now, in ΔABC and ΔPQR, we obtain:
AB = PQ (By construction)
BC = QR (By construction)
AC = PR [From (3)]

Therefore, by SSS congruency criterion, $\Delta ABC \cong \Delta PQR$

$\therefore \angle B = \angle Q$ (By CPCT)
However, $\angle Q = 90°$ (By construction)
$\therefore \angle B = 90°$

Hence, in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Now, we draw ΔPQR as:



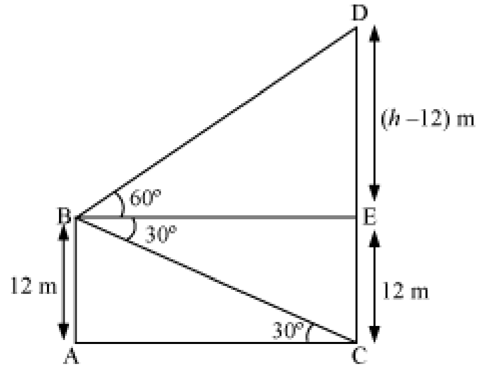
It is given that in ΔPQR:
PQ = QR … (1)

Also given that $PR^2 = 2PQ^2$
$\Rightarrow PR^2 = PQ^2 + PQ^2$
$\Rightarrow PR^2 = PQ^2 + (QR)^2$ {From (1)}
$\Rightarrow PR^2 = PQ^2 + QR^2$

Thus, by applying the theorem proved above, $\angle Q$ is a right angle.

28. A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3} = 1.732$]

Solution:
The given situation can be represented diagrammatically as:



Here, AB represents the deck of the ship above water level, DC represents the cliff, and AC represents the water level. Point B represents the position of the man.
According to the given condition, AB = 12 m, ∠DBE = 60°, ∠EBC = 30°, which implies that ∠BCA = 30°.
Also, CE = AB = 12 m
Let CD = *h* m
Then, ED = (*h* – 12) m

In right triangle CAB:

$$\frac{AB}{AC} = \tan 30°$$

$$\Rightarrow \frac{12 \text{ m}}{AC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AC = 12\sqrt{3} \text{ m}$$

$$\therefore \ BE = 12\sqrt{3} \text{ m} \qquad \ldots (1)$$

In right triangle BED:

$$\frac{DE}{BE} = \tan 60°$$

$$\Rightarrow \frac{(h-12) \text{ m}}{12\sqrt{3} \text{ m}} = \sqrt{3} \qquad \text{[Using (1)]}$$

$$\Rightarrow (h-12) = 12\sqrt{3} \times \sqrt{3} \text{ m} = 36 \text{ m}$$

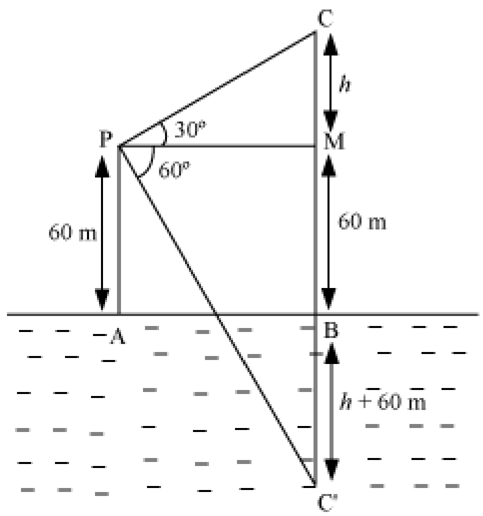
$$\Rightarrow h = (36+12) \text{ m} = 48 \text{ m}$$

∴ Height of the cliff = 48 m
Distance of the cliff from the ship = $12\sqrt{3}$ m = (12 × 1.732) m = 20.784 m

OR

The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud from the surface of the lake.

Solution:
Let AB be the surface of the lake and P be the point of observation such that AP = 60 m.
Let C be the position of the cloud and C′ be its reflection in the lake.
Then CB = C′B
Draw PM ⊥ CB
Let CM = *h*
∴ CB = *h* + 60 m



In ΔCPM:

$$\tan 30° = \frac{CM}{PM}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PM}$$

$$\Rightarrow PM = \sqrt{3}h \qquad ...(1)$$

In ΔPMC′:

$$\tan 60° = \frac{C'M}{PM}$$

$$\Rightarrow \tan 60° = \frac{C'B + BM}{PM}$$

$$\Rightarrow \sqrt{3} = \frac{h + 60\text{ m} + 60\text{ m}}{PM}$$

$$\Rightarrow PM = \frac{h + 120\text{ m}}{\sqrt{3}} \qquad ...(2)$$

From equations (1) and (2):

$\sqrt{3}h = \frac{h + 120\,\text{m}}{\sqrt{3}}$

$\Rightarrow 3h = h + 120\,\text{m}$

$\Rightarrow 2h = 120\,\text{m}$

$\Rightarrow h = 60\,\text{m}$

CB = h + 60 m = 60 m + 60 m = 120 m

Thus, the height of the cloud from the surface of the lake is 120 m.

29. The surface area of a solid metallic sphere is 616 cm^2. It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution:
The surface area of the solid metallic sphere is given as 616 cm^2.
Let r be the radius of the sphere.

$\therefore\ 4\pi r^2 = 616\ \text{cm}^2$

$\Rightarrow r^2 = \frac{616}{4\pi}\ \text{cm}^2 = \frac{616 \times 7}{4 \times 22}\ \text{cm}^2 = 49\ \text{cm}^2$

$\Rightarrow r = 7\ \text{cm}$

It is given that the sphere is melted and is recast into a cone of height 28 cm.
The volume remains the same in case of conversion of solids from one shape to another.

$\therefore$ Volume of the sphere = volume of the cone

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 H$, where R and H are respectively the radius and height of the cone

$\Rightarrow \frac{4}{3}(7)^3 = \frac{1}{3}R^2(28)$

$\Rightarrow R^2 = \frac{4 \times 7 \times 7 \times 7}{28} = (7)^2$

$\Rightarrow R = 7\ \text{cm}$

$\therefore$ Diameter = 2 × R = 2 × 7 cm = 14 cm

Thus, the diameter of the base of the cone so formed is 14 cm.

OR

The difference between the outer and inner curved surface areas of a hollow right circular cylinder, 14 cm long, is 88 cm^2. If the volume of metal used in making the cylinder is 176 cm^3, find the outer and inner diameters of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution:
Height of the cylinder, h = 14 cm
Let R and r be the outer and inner radii of the hollow cylinder.

It is known that, curved surface area of cylinder = $2\pi \times$ radius $\times$height
Therefore, according to the given information:
$2\pi Rh - 2\pi rh = 88$ cm^2
$\Rightarrow 2\pi\,(14)\,(R - r) = 88$
$\Rightarrow R - r = \frac{88 \times 7}{2 \times 22 \times 14} = 1$ ...(1)

It is also given that the volume of the metal used in making the cylinder is 176 cm^3.
$\therefore \pi(R^2 - r^2)h = 176$
$\Rightarrow \frac{22}{7} \times 14 \times (R + r)(R - r) = 176$
$\Rightarrow 44 \times (R + r)(1) = 176$ [Using (1)]
$\Rightarrow (R + r) = \frac{176}{44} = 4$
$\Rightarrow R + r = 4$ ...(2)
Adding (1) and (2):
$2R = 5$
$R = \frac{5}{2}$ cm
$r = \left(4 - \frac{5}{2}\right)$ cm $= \frac{3}{2}$ cm

Thus, the outer diameter of the cylinder is $2R = 5$ cm, and its inner diameter is $2r = 3$ cm

30. Draw 'less than ogive' and 'more than ogive' for the following distribution and hence find its median.

Class	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Frequency	8	12	24	6	10	15	25

Solution:
To draw the less than ogive and the more than ogive of the given frequency distribution, we need to construct the less than type and more than type cumulative frequency tables first.
This can be done as:

Less than type cumulative frequency table:

Class	Upper class limit	Frequency	Cumulative frequency
20 – 30	30	8	8
30 – 40	40	12	8 + 12 = 20
40 – 50	50	24	20 + 24 = 44
50 – 60	60	6	44 + 6 = 50
60 – 70	70	10	50 + 10 = 60
70 – 80	80	15	60 + 15 = 75
80 – 90	90	25	75 + 25 = 100

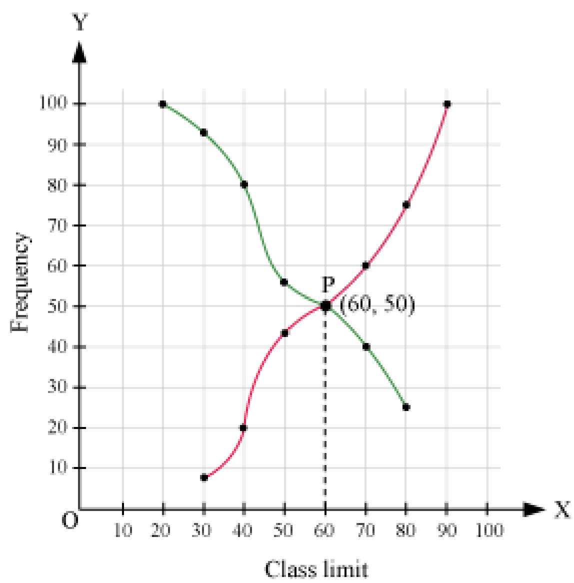
We take the horizontal axis as the upper class limit and the vertical axis as the corresponding cumulative frequency, and plot the cumulative frequency for each upper class limit to obtain the required ogive (of less-than type).

More than type cumulative frequency table:

Class	Lower class limit	Frequency	Cumulative frequency
20 – 30	20	8	92 + 8 = 100
30 – 40	30	12	80 + 12 = 92
40 – 50	40	24	56 + 24 = 80
50 – 60	50	6	50 + 6 = 56
60 – 70	60	10	40 + 10 = 50
70 – 80	70	15	25 + 15 = 40
80 – 90	80	25	25

We take the horizontal axis as the lower class limit and the vertical axis as the corresponding cumulative frequency, and plot the cumulative frequency for each lower class limit to obtain the required ogive (of more-than type).

The less than type and more than type ogives can be drawn on graph paper as:



It can be observed that the less than type and more than ogives of the given frequency distribution intersect at point P (60, 50). If we draw a perpendicular to the *x*-axis from point P, then the value on the *x*-axis is obtained as 60.

Thus, the median of the given data is 60.